Quantum Control via Geometrical Optimization

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The selective preparation of arbitrary states by means of laser protocols is a paramount goal of quantum control, one that has primary importance in quantum engineering, particularly in quantum information or quantum computation processes. Quantum optimal control theory is especially well designed to find the optimal pulses. There are general quantum controllability theorems that can be used to non-constructively establish the feasibility of the enterprise while the study of the quantum landscapes can provide additional details regarding the topological features of the optimal solutions. In this work we will move between quantum control designs and quantum controllability, analyzing the general features of some quantum control schemes assuming certain constrained controllability criteria. To that end we will use the Rayleigh-Ritz variational approach applied not to Hermitian operators (observables) whose value is minimized, but to the time-evolution operator, maximizing transition probabilities. This approach is equivalent to a geometrical optimization of the dynamics where the time evolution operator is replaced by an ordinary rotation in a subset of the Hilbert space.

In particular, we will be concerned with intrinsic properties of the dynamics of systems with manifolds of sublevels, that we call quantum substructures, which pose several interesting problems from the point of view of controlling the system dynamics. Quantum control typically implies the ability to manipulate interfering pathways. Increasing the number of levels that participate in the dynamics, a multilevel structure would therefore offer more control opportunities at the expense of the ability to manipulate within the substructure. We will focus on a relatively general quantum control scheme based on parallel transfer. However, as we will demonstrate, substructures often give raise to Stark effects or Autler-Townes splittings that actively block population transfer. A balance must therefore take place between the number of levels that participate in the dynamics and those that can be controlled and to what extent. Defining different ways to set up this balance, our general goal is to investigate whether quantum structures limit, or conversely help, in controlling the system.

Studying population transfer between sub-manifolds we will show that coarse grain population passage can be made faster via parallel transfer, and by suitably preparing particular initial superposition states one can almost overcome the tyranny of the Rabi area, increasing the controllability of the system as long as all initial sublevels can be accessed. On the other hand, in preparing arbitrary superposition states one needs to have full controllability on to both the initial and the final manifold of sublevels.