

Probing BSM physics with isotope shifts

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CNRS/LAPTH

FRANCE

- CD, Soreq, in progress
- CD, Frugiuele, Fuchs, Soreq, PRD (2017)
- Berengut, Budker, CD, Flambaum, Frugiuele, Fuchs, Grojean, Harnik, Ozeri, Perez, Soreq, hep-ph/1704.05068
- CD, Ozeri, Perez, Soreq, PRD 96 (2017) 093001



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Why BSM at low energies?

- ❑ Agnostic: why not?

- ❑ SM hierarchy problems:

- Strong CP → light axion particle: $m_a \sim 10^{-6}$ eV
actively searched through $\frac{1}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu}$

- Higgs mass: $\mu^2 H^\dagger H$ with $\mu^2 \sim \Lambda^2$ at quantum level
which (used to?) motivate BSM at $\Lambda \sim \text{TeV}$.
Yet there is a (first?) counter-example: relaxion

- ❑ New physics could show up at **any** scale!

time to join efforts at high and low energy frontiers

Original motive: Higgs

- Higgs boson was discovered. Yet little is known about its couplings to fermions; SM predicts $y_f = m_f/v$.

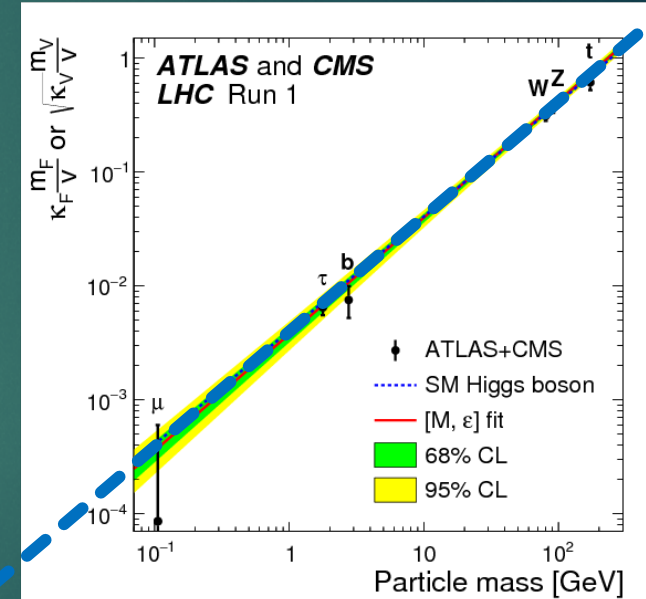
- High-energy colliders tell us about heavy fermions: t, b, τ, μ (c?)

- Lighter u, d, s, e fermions are very challenging!

$$y_{u,d,s} < y_b \quad \text{Perez+ PRD (2016)}$$

$$y_e < 3y_\mu \quad \text{Altmannshofer+ JHEP (2015)}$$

- Maybe atomic probes are better?



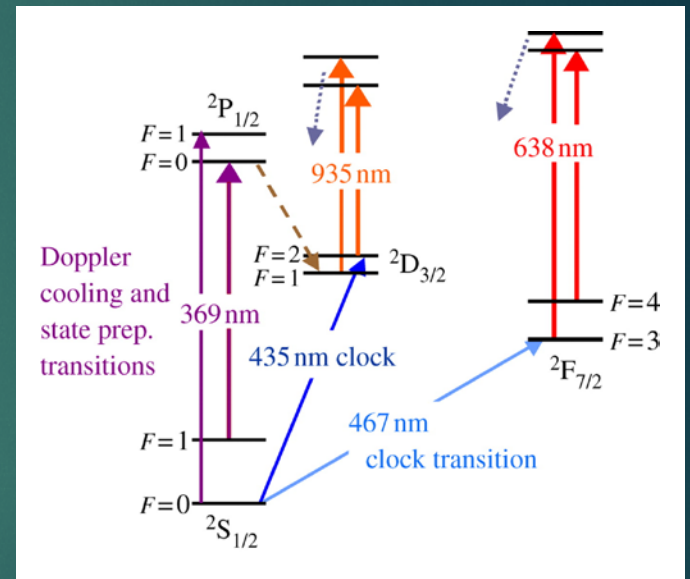
e
 u
 d

- eg. Ytterbium ion
- Godun+ PRL (2014)
Huntermann+ PRL (2014)
- $\nu_3 = 642\,121\,496\,772\,645.36(25) \text{ Hz}$
- (shift from Earth's gravity pull -0.046 Hz)

Huntermann+ PRL (2014)

$$\nu_{E3} = 642\,121\,496\,772\,645.36(25) \text{ Hz}$$

(shift from Earth's gravity pull -0.046 Hz)



- E3 is stable at $\sim 10^{-18}$ level Huntermann+ PRL (2016)

Towards BSM probes

- ❑ In principle sensitive to BSM effects as small as $\text{QED}/10^{16}$
- ❑ However probing BSM further requires either:
 - ❑ Precise QED calculation – only available for atoms/ions with 1,2 (maybe 3) electrons: H, He...
 - ❑ Combining measurements and reduce sensitivity to uncertain quantities from theory: isotope shifts, King linearity

BSM atomic potential

- Consider a **new boson** ϕ with P-conserving couplings to electron and nucleons:

The diagram shows the equation for the BSM atomic potential $V_\phi(r)$ with four orange arrows pointing to specific parts of the formula:

- An arrow from "mediator spin" points to the term $(-1)^{s+1}$.
- An arrow from "mediator mass" points to the term m_ϕ in the exponent of the exponential function.
- An arrow from "electronic coupling" points to the term y_e .
- An arrow from "nuclear coupling" points to the term y_A .

Below the nuclear coupling label, the expression for y_A is given:

$$y_A = Z y_p + (A - Z) y_n$$

Isotope shift

- QED effects cancel between isotopes A, A' up to:

$$\nu_i^{AA'} = K_i \mu_{AA'} + F_i \delta \langle r^2 \rangle_{AA'}$$

mass shift (MS)

$$\mu_{AA'} \equiv (m_A^{-1} - m_{A'}^{-1})$$

field shift (FS)

- For odd A , there are also nuclear spin effects
- BSM effects mildly suppressed by $(A-A')/A \sim 0.1$:

$$\nu_i^{AA'} \Big|_{\text{BSM}} \simeq \alpha_{\text{NP}} (A - A') X_i$$

$$\alpha_{\text{NP}} \equiv \frac{(-1)^{s+1} y_e y_n}{4\pi}$$

- K, F, X are electronic constant, independent of A at LO

Isotope shifts in Helium

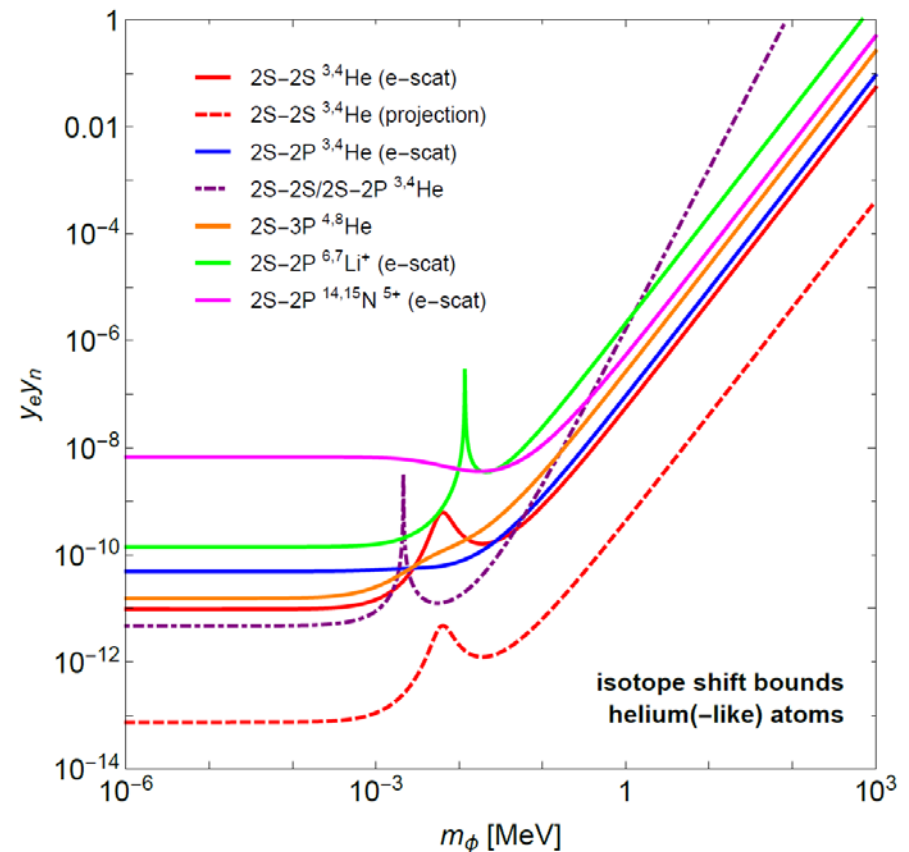
- Helium 3,4 IS theory calculations (for point nuclei) are known better than experimental error: [Pachucki+ PRA \(2017\)](#)

- Nuclear radii are known from scattering:

$$\delta\langle r^2\rangle_{3,4} = 1.067(65)$$

- Combining 2 transitions to eliminate $\delta\langle r^2\rangle$ helps
- Expected improvement by a factor ~ 100 with μHe^+ measurements

[Antognini+ Can. J. Phys. \(2011\)](#)



Isotope shifts in HD

- Despite proton radius puzzle, electronic and muonic values of $\delta\langle r^2\rangle_{\text{HD}}$ are consistent.

- From muonic Lamb shifts:

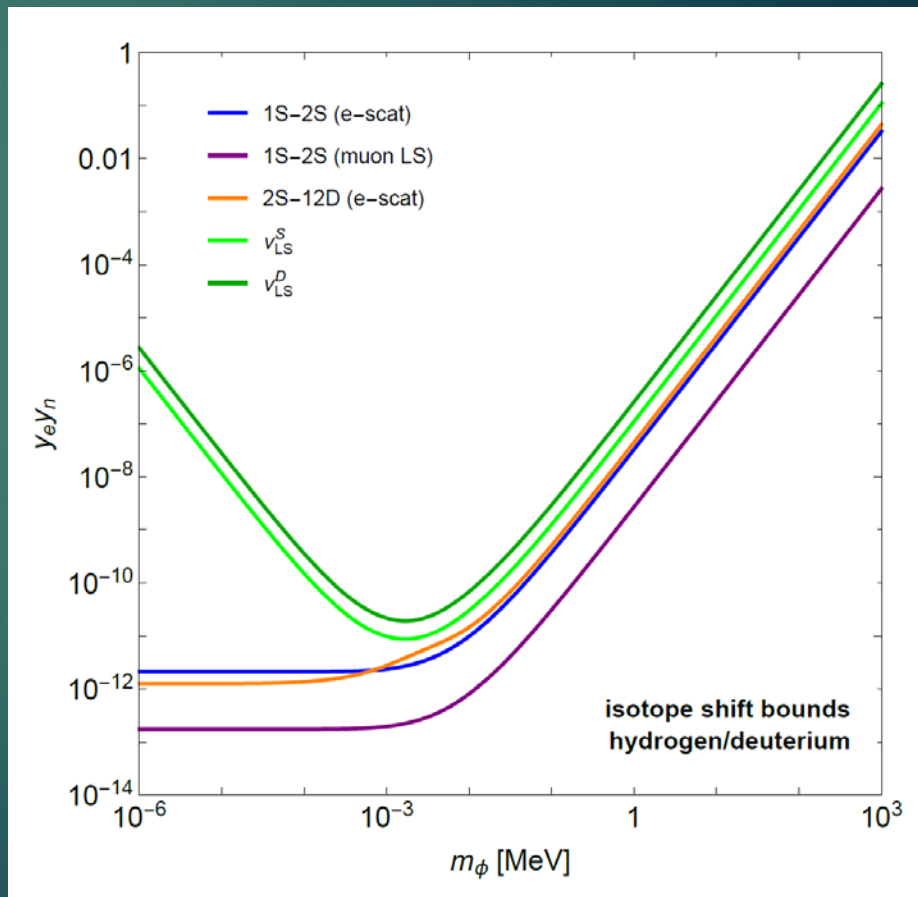
$$\delta\langle r^2\rangle_{\mu\text{HD}} = 3.8112(34)$$

Pohl+ Nature (2010), Antognini+ Science (2013)
Pohl+ Science (2016)

- Theory error dominates, limited by m_e/m_D , nuc.pol.

Parthey+ PRL (2010)

- HD sensitivity comparable to Helium



Heavy atoms?

- ❑ In atoms with many electrons **e-e correlation effects** are not predictable from theory to sufficient accuracy
→ direct TH/EXP comparison not possible...
- ❑ Is there any observable sensitive to BSM and limited only by experimental uncertainty?
→ **King linearity**
King J. Opt. Soc. Am. (1963)
- ❑ Basic idea = combine IS measurements in 2 transitions with many (at least 4) isotopes

King linearity

- In the limit that electronic and nuclear parameters are **factorized** as $\nu_i^{AA'} = K_i \mu_{AA'} + F_i \delta \langle r^2 \rangle_{AA'}$ there is a linear relation between IS in 2 transitions.
- Defining « modified IS » as $m\nu_i^{AA'} \equiv \mu_{AA'}^{-1} \nu_i^{AA'}$, one finds:

$$m\nu_2^{AA'} = F_{21} m\nu_1^{AA'} + K_{21}$$

slope = F_2/F_1

offset = $K_2 - F_{21}K_1$

Establishing King linearity from data

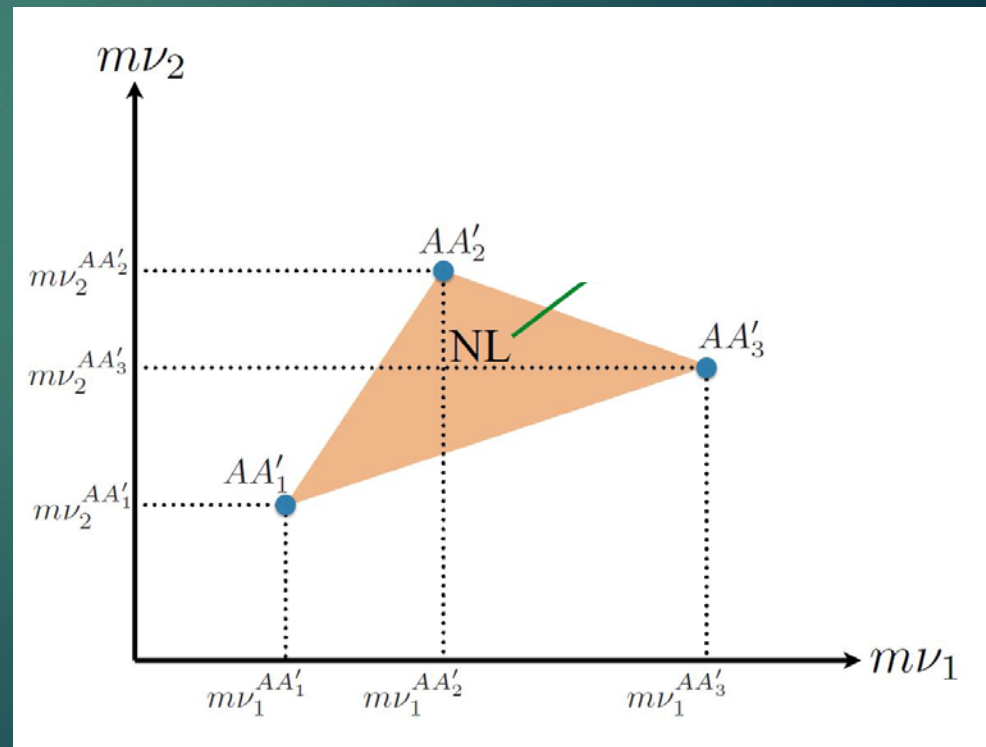
- Need 3 points on King plot → 2 transitions, 4 (even) isotopes
- Invariant measure of non-linearities is the **triangle area**:

$$NL = \frac{1}{2} \left| \overrightarrow{m\nu_1} \times \overrightarrow{m\nu_2} \cdot \vec{1} \right|$$

$$\overrightarrow{m\nu_{1,2}} \equiv \left(m\nu_{1,2}^{AA'_1}, m\nu_{1,2}^{AA'_2}, m\nu_{1,2}^{AA'_3} \right)$$

$$\vec{1} \equiv (1, 1, 1)$$

- If $NL \lesssim \sigma_{NL}$ then the King plot is linear



BSM effects break linearity

- In the presence of a BSM force:

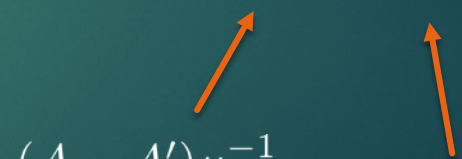
$$\nu_i^{AA'} = K_i \mu_{AA'} + F_i \delta \langle r^2 \rangle_{AA'} + \alpha_{\text{NP}} (A - A') X_i$$

- Combining 2 transitions to eliminate $\mu_{AA'}^{-1}, \delta \langle r^2 \rangle_{AA'}$ yields:

$$m\nu_2^{AA'} = F_{21} m\nu_1^{AA'} + K_{21} + \alpha_{\text{NP}} h_{AA'} X_{21}$$

- Non-linearities from BSM unless:

- $X_{21} \rightarrow 0$, ie. for short-range forces
- $h_{AA'} \propto m\nu_i^{AA'}$ or constant of AA'

$$(A - A') \mu_{AA'}^{-1} \quad X_2 - F_{21} X_1$$


Bounding BSM coupling

- Manipulating vectors:

$$\text{NL}_{\text{NP}} = \frac{\alpha_{\text{NP}}}{2} (\vec{1} \times \vec{h}) \cdot (X_1 \overrightarrow{m\nu_2} - X_2 \overrightarrow{m\nu_1})$$

- So for a given (linear) data-set $\overrightarrow{m\nu_1}, \overrightarrow{m\nu_2}$ we can bound

$$\alpha_{\text{NP}} \leq \frac{(\overrightarrow{m\nu_1} \times \overrightarrow{m\nu_2}) \cdot \vec{1}}{(\vec{1} \times \vec{h}) \cdot (X_1 \overrightarrow{m\nu_2} - X_2 \overrightarrow{m\nu_1})}$$

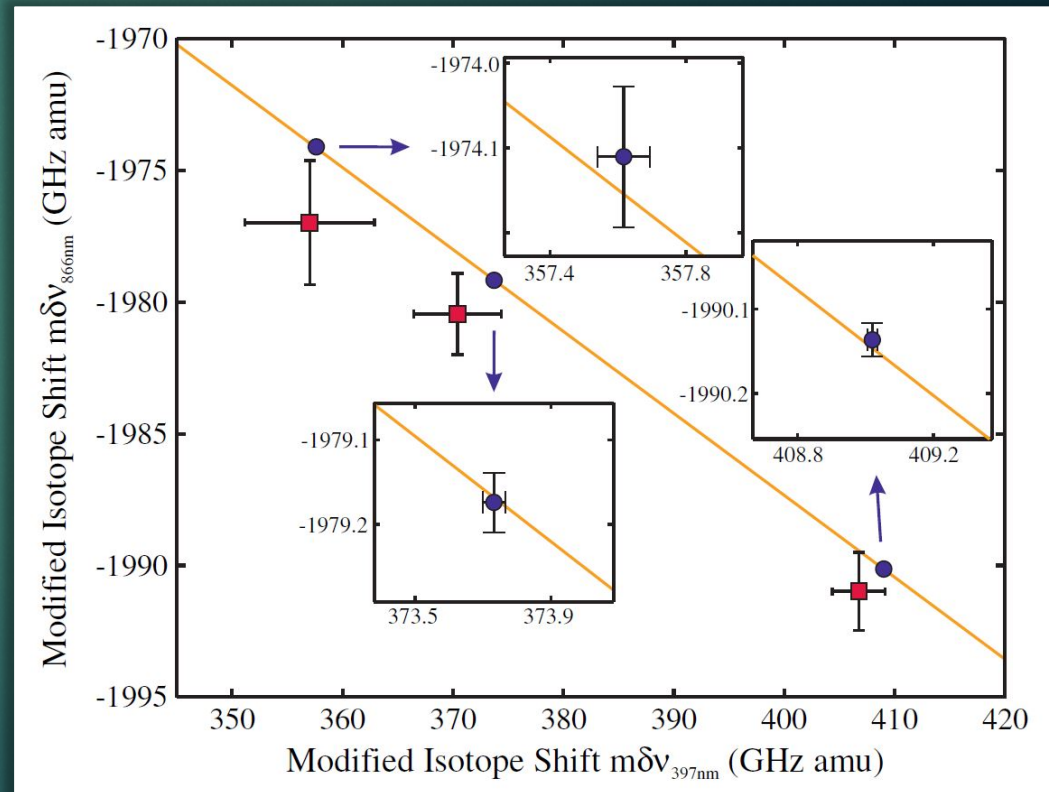
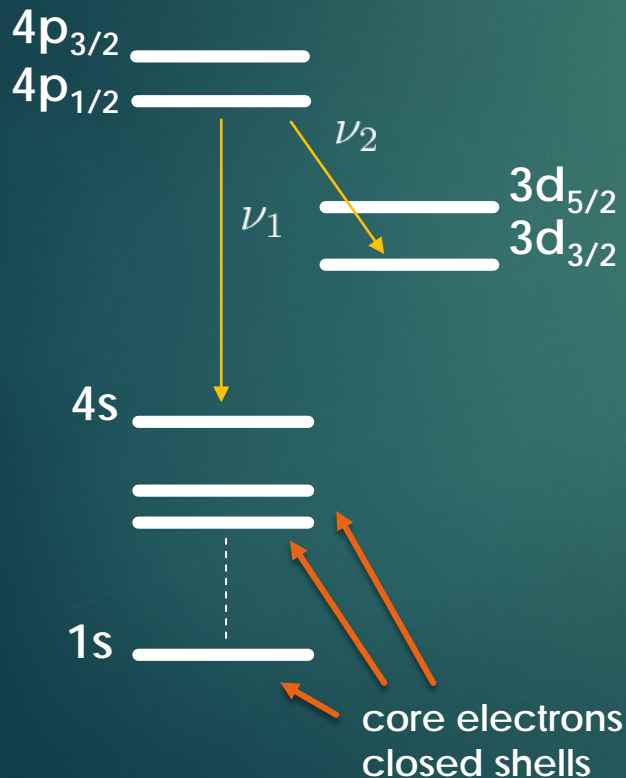
the **only theoretical inputs** (depend on m_ϕ)
calculated using many-body perturbation theory

King linearity in data

□ **Calcium+** $A=40,42,44,48$

precision $\sim 100\text{kHz}$

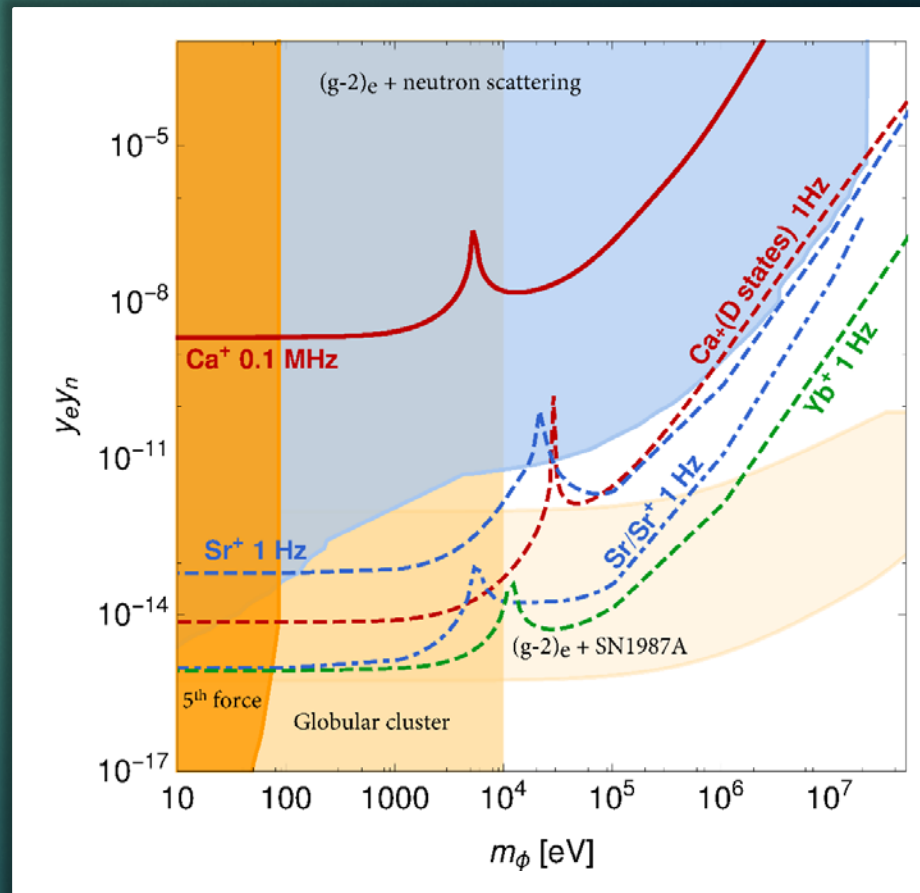
Gebert+ PRL (2015)



linearity tested to $\sim 10^{-4}$

Bounds and projections

- Ca⁺ bound weaker than constraints from other sources:
 - neutron scattering,
 - electron g-2,
 - star cooling
 - ...
- Projected sensitivity of clock transitions in several elements: eg. Sr, Sr⁺, Yb⁺ with Hz accuracy could explore new territory
- Need linearity to hold upto $\sim 10^{-9}$



One King to rule them all (preliminary)

- Higher order nuclear effects also induce non-linearities:

$$\nu_i^{AA'} = K_i \mu_{AA'} + F_i \delta \langle r^2 \rangle_{AA'} + \sum_{j=1}^{n-2} G_{ij} O_{AA'}^j + \alpha_{\text{NP}} (A - A') X_i$$

- One can either calculate them (again hard...) or **use more measurements to remove more « spurions »**
- For n-2 spurions, need n transitions with n+1 isotope pairs.
- w/out BSM, the n IS vectors are on a plane in n+1 dimensions → King planarity
- w/ BSM IS vectors form a volume in n+1 dimensions
- For planar data,

$$\alpha_{\text{NP}} \leq \frac{\epsilon_{A_1 \dots A_{n+1}} m \nu_{1 A_1} \dots m \nu_{n A_n}}{-\epsilon_{A_1 \dots A_{n+1}} \epsilon_{i_1 \dots i_n} X_{i_1} h_{A_1} \dots m \nu_{i_n, A_{n-1}} / (n-1)!}$$

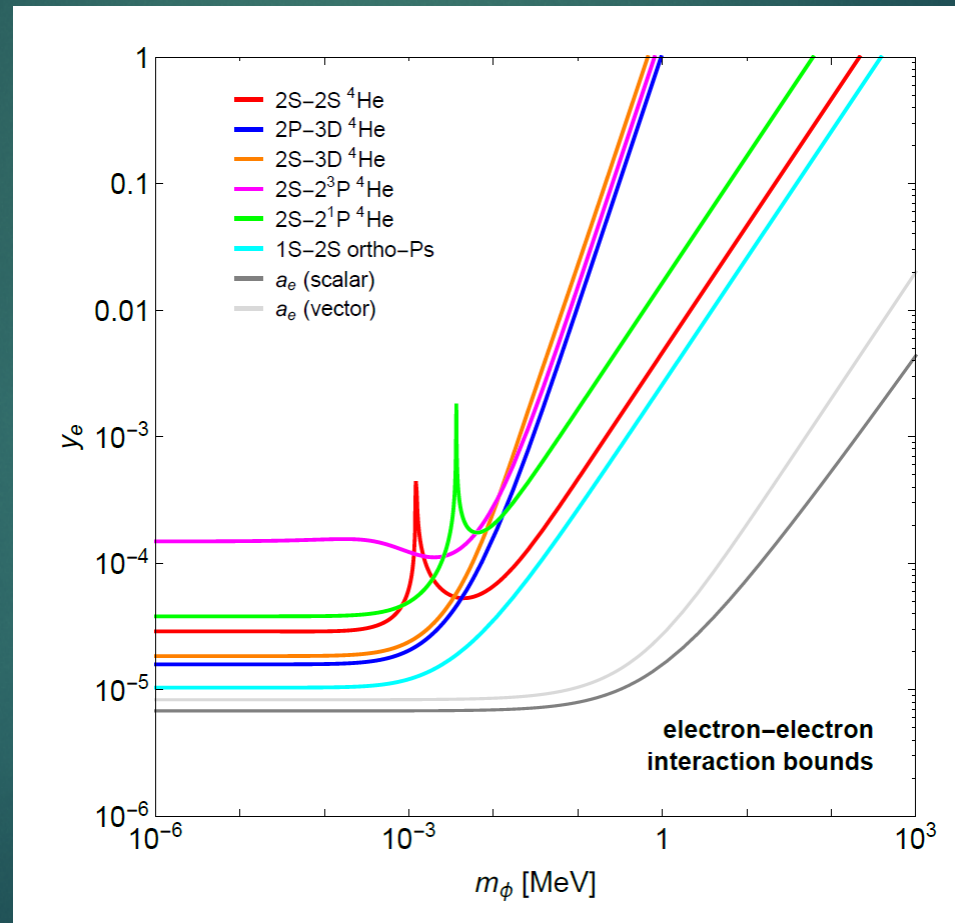
Conclusion, Outlook

- ❑ New physics can be anywhere. Need to measure whatever is possible..
- ❑ Isotope shifts and **King linearity** is an effective probe **only limited by experimental errors!**
- ❑ So far focused on spin-independent forces, **are there similar observables sensitive to spin-dependent forces?**

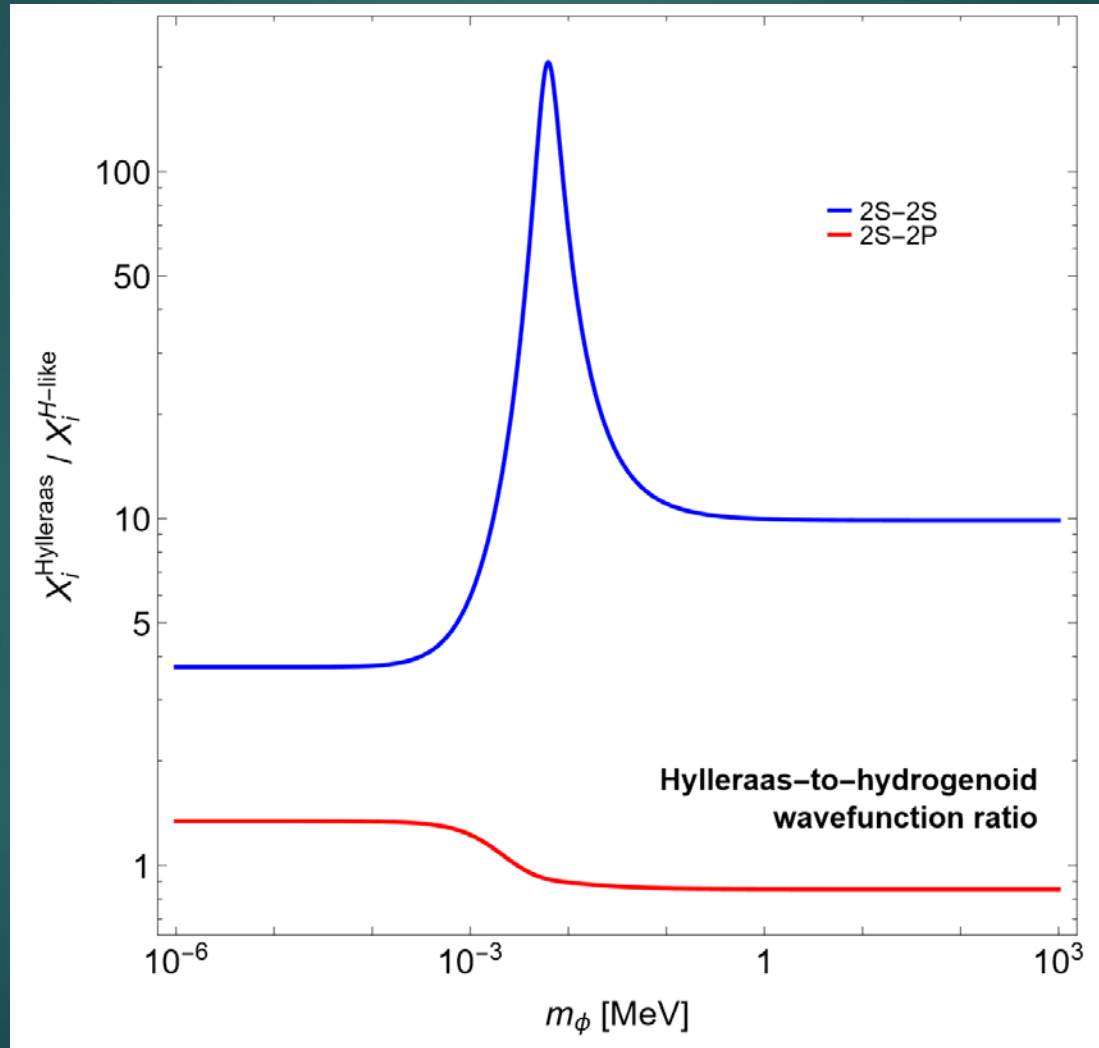


Backups

Electron-electron interactions with Helium 4



Hylleraas wavefunctions



Sensitivity estimates

- ▶ Assume linearity is established within a precision Δ
- ▶ Then, factorization holds within uncertainties:

$$\overrightarrow{m\nu}_i = K_i \vec{1} + F_i \overrightarrow{\delta \langle r^2 \rangle} + \overrightarrow{\Delta}_i$$

- ▶ And a best-case estimate of the resulting bound is:

$$\alpha_{\text{NP}} \leq \frac{\sqrt{\Delta_1^2 + F_{21} \Delta_2^2}}{(X_2 - F_{21} X_1)(A - A')_{\text{max}}} \times \frac{A}{(A - A')_{\text{min}}}$$

suppression factor
for short-range forces

calculated
from MBPT
Berengut+ PRA (2006)

alignment with MS