Isotope shift in the search for nuclear island of stability and new particles

V. V. Flambaum, A. J. Geddes, A. V. Viatkina

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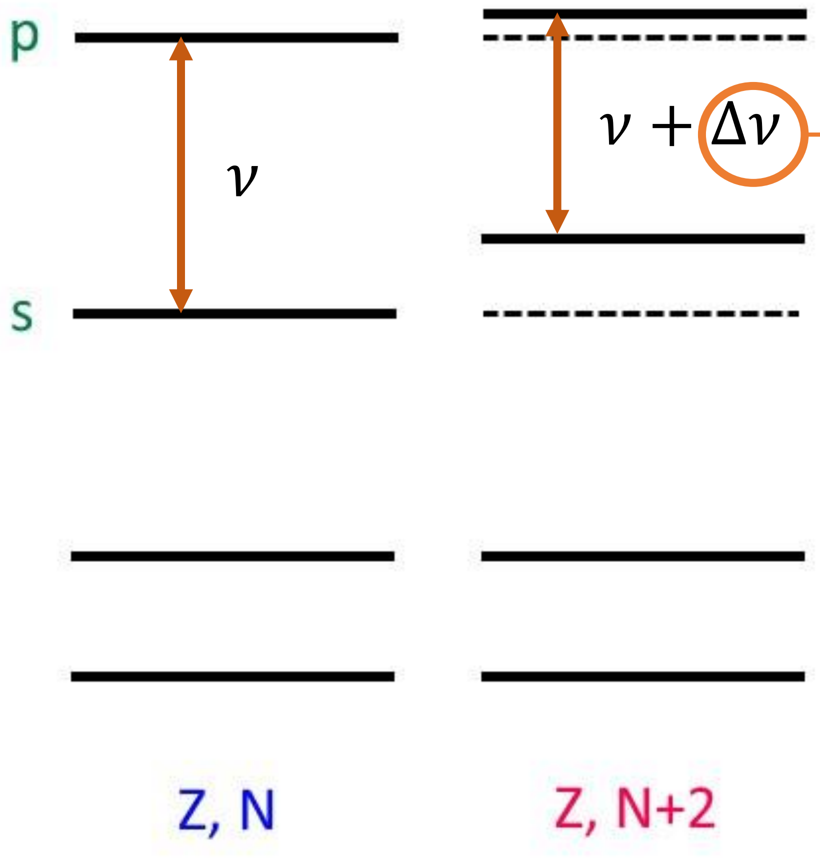
BSM in direct, indirect and tabletop experiments

Weizmann Institute of Science, Israel

Isotope shift

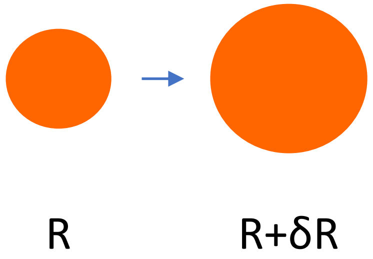


$$\Delta\nu = IS = MS + FS$$

Normal mass shift (NMS)
center of mass change
(finite nuclear mass effect)

Specific mass shift (SMS)
electron correlation

Field shift
change of the
nuclear electric
charge distribution



King plot

$$IS = MS + FS$$

Let's look at a pair of transitions in two isotopes A and A'.
First-order non-relativistic case gives:

MS FS

$$\Delta\nu_{1,AA'} = K_1 \cdot \mu_{AA'} + F_1 \cdot \delta\langle r^2_{AA'}\rangle$$

$$\Delta\nu_{2,AA'} = K_2 \cdot \mu_{AA'} + F_2 \cdot \delta\langle r^2_{AA'}\rangle$$

, here $\mu_{AA'} = \frac{1}{M_A} - \frac{1}{M_{A'}}$

electronic nuclear

$$n_{1,AA'} = {\Delta\nu_{1,AA'}}/{\mu_{AA'}} = K_1 + F_1 x_{AA'}$$

$$n_{2,AA'} = {\Delta\nu_{2,AA'}}/{\mu_{AA'}} = K_2 + F_2 x_{AA'}$$

, $x_{AA'} = \frac{\delta\langle r^2_{AA'}\rangle}{\mu_{AA'}}$

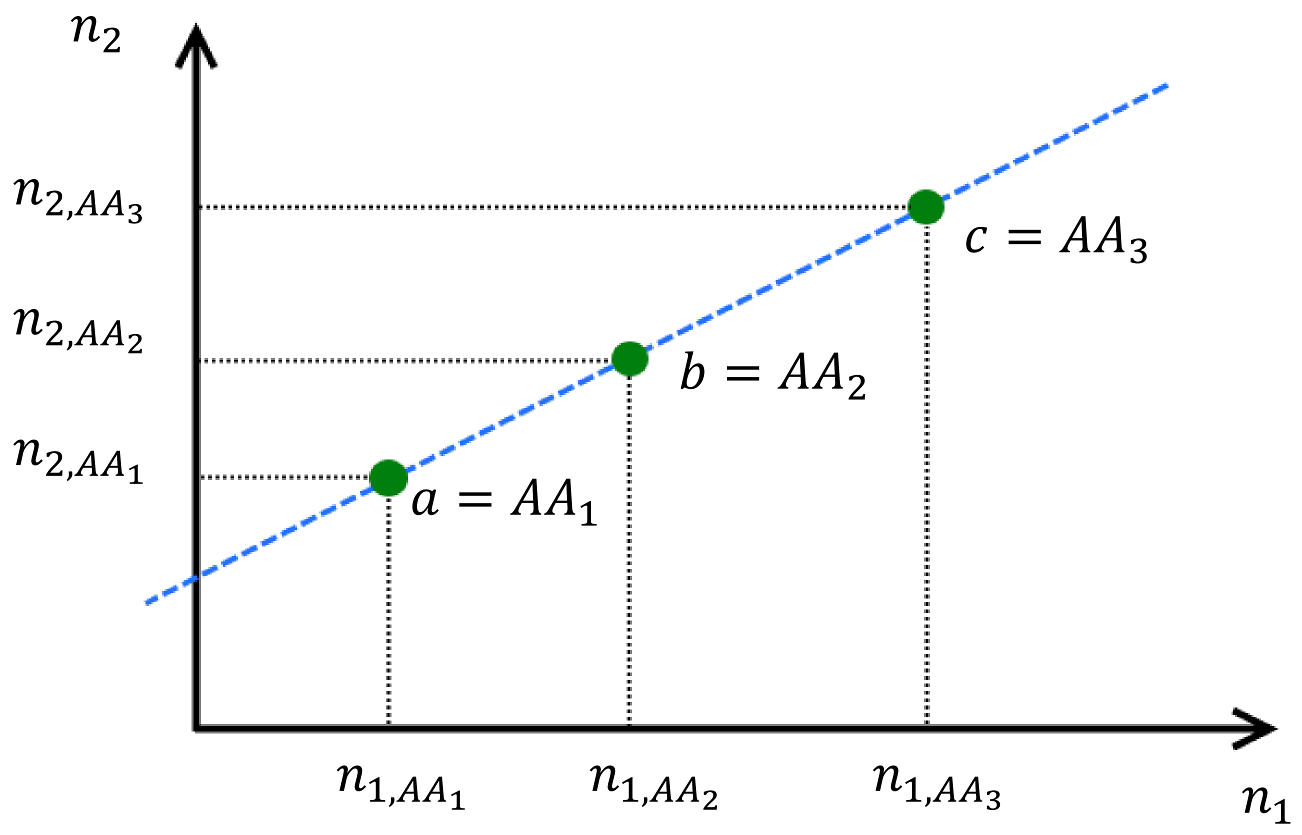
$$n_{2,AA'} = K_2 + \frac{F_2}{F_1}(n_{1,AA'} - K_1)$$

[1] King, W. H. (1963)

King plot

$$n_{2,AA'} = K_2 + \frac{F_2}{F_1}(n_{1,AA'} - K_1)$$

A, A_1, A_2, A_3 - isotopes of an element Z



King plot non-linearity

$$IS = MS + FS$$

$$\mu_{AA'} = \frac{1}{M_A} - \frac{1}{M_{A'}}$$

$$\Delta\nu_{1,AA'} = K_1 \mu_{AA'} + F_1 \delta\langle r_{AA'}^{2\gamma_1}\rangle \boxed{+ G_1 \delta\langle r_{AA'}^{2\gamma_2}\rangle + \cdots}$$

$$\Delta\nu_{2,AA'} = K_2 \mu_{AA'} + F_2 \delta\langle r_{AA'}^{2\gamma_1}\rangle \boxed{+ G_2 \delta\langle r_{AA'}^{2\gamma_2}\rangle + \cdots}$$

$$\gamma_1 = \sqrt{{\kappa_1}^2 - (\alpha Z)^2}$$

$$\kappa_1 = -1, 1$$

$$\gamma_2 = \sqrt{{\kappa_2}^2 - (\alpha Z)^2}$$

$$\kappa_2 = -2, 2, -3, 3, \ldots$$

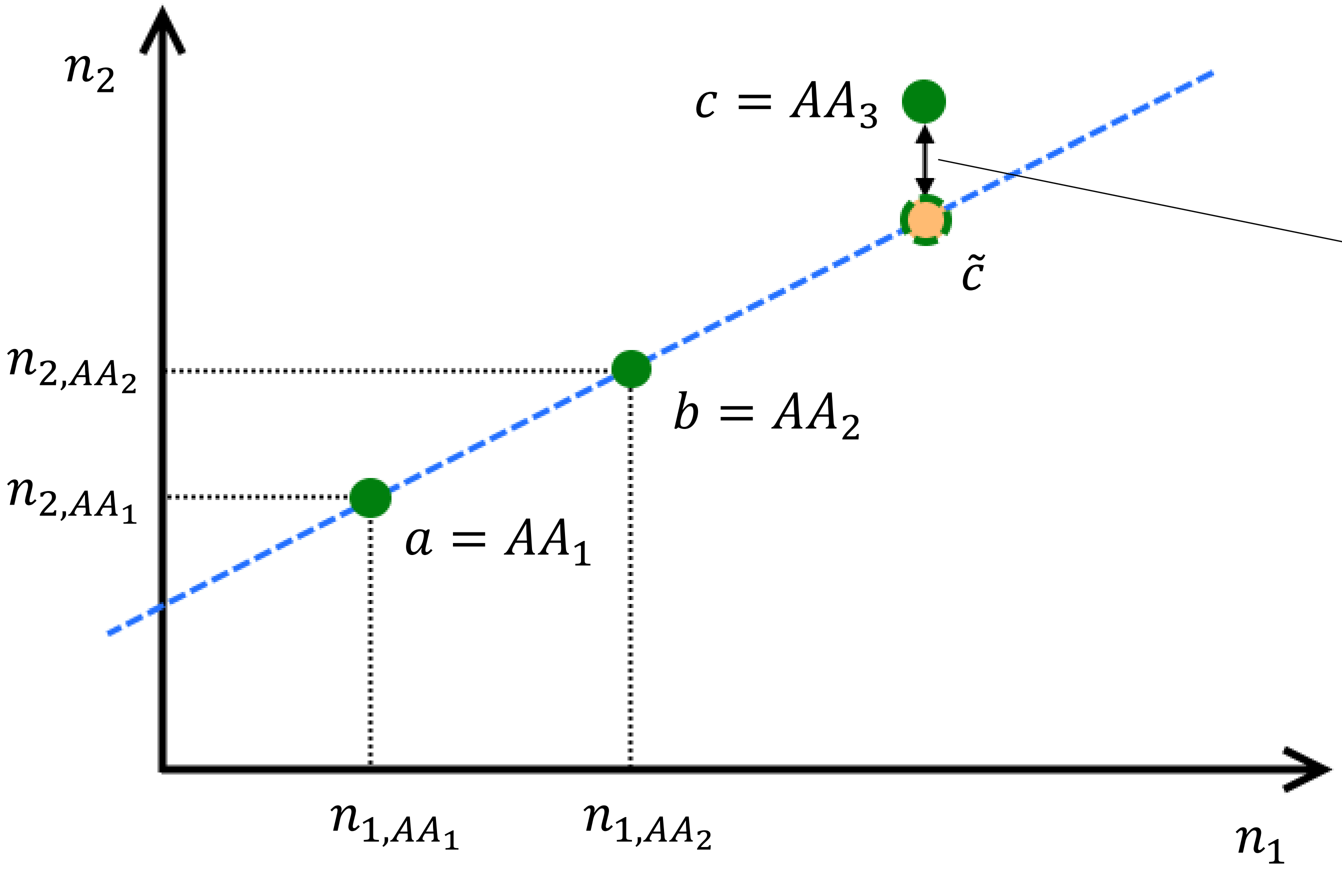
divided both by $\mu_{AA'}$

$$n_{1,AA'} = K_1 + F_1 x_{AA'} \boxed{+ G_1 y_{AA'} + \cdots}$$

$$n_{2,AA'} = K_2 + F_2 x_{AA'} \boxed{+ G_2 y_{AA'} + \cdots}$$

???

King plot non-linearity



- **Sources of nonlinearities from higher-order SM contributions ?**

or

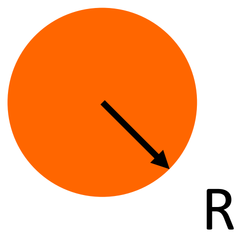
- **New Physics ?**

Higher-order SM contributions

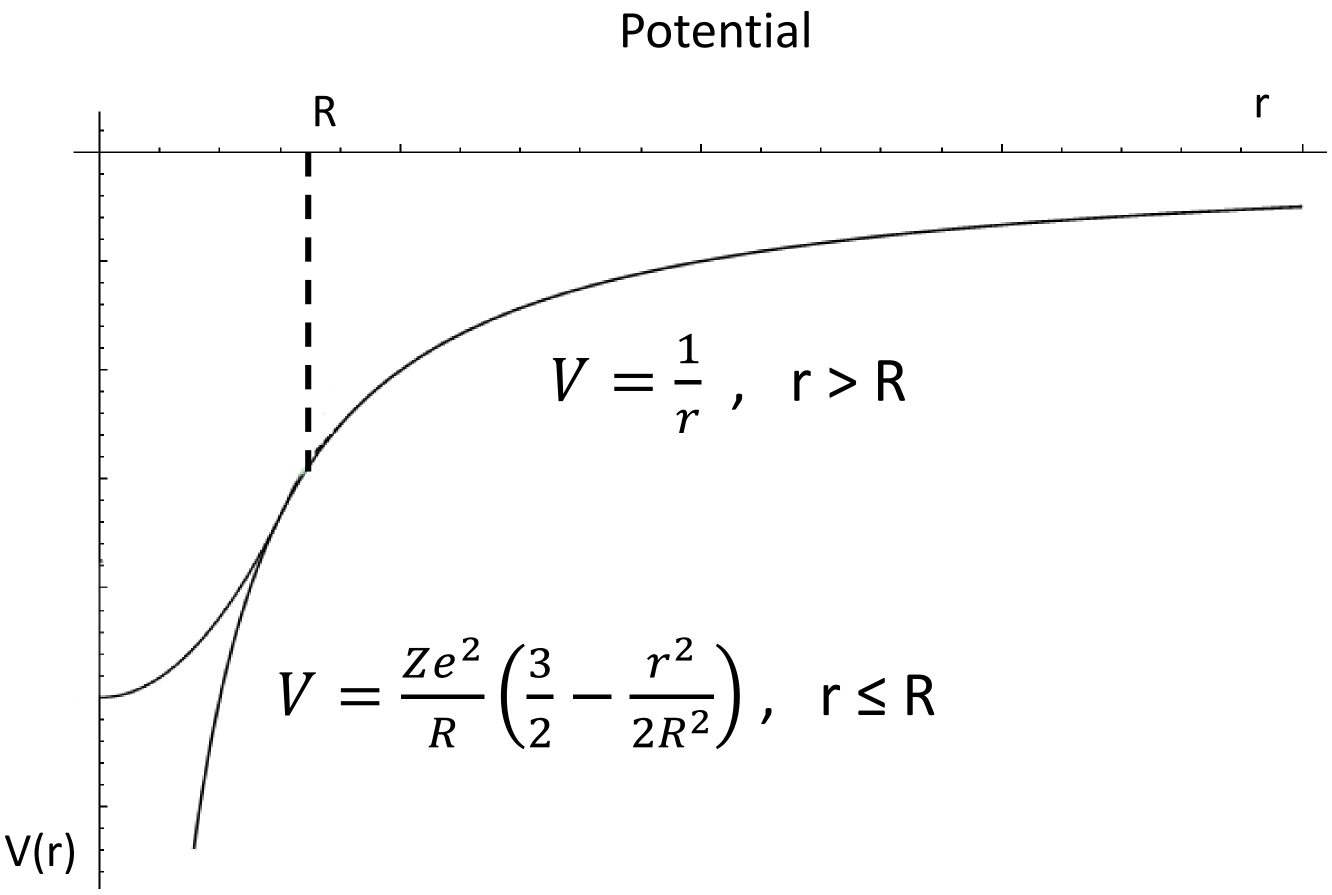
- FS affected by higher waves in the transition ($p_{3/2}$, $d_{3/2}$, $d_{5/2}$ …)
- Nuclear polarizability
- Many-body effects
- …

Field shift

single-electron mean-field approximation

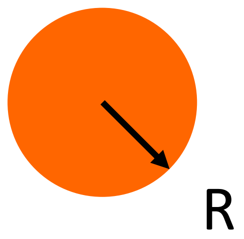


Uniformly charged spherical nucleus



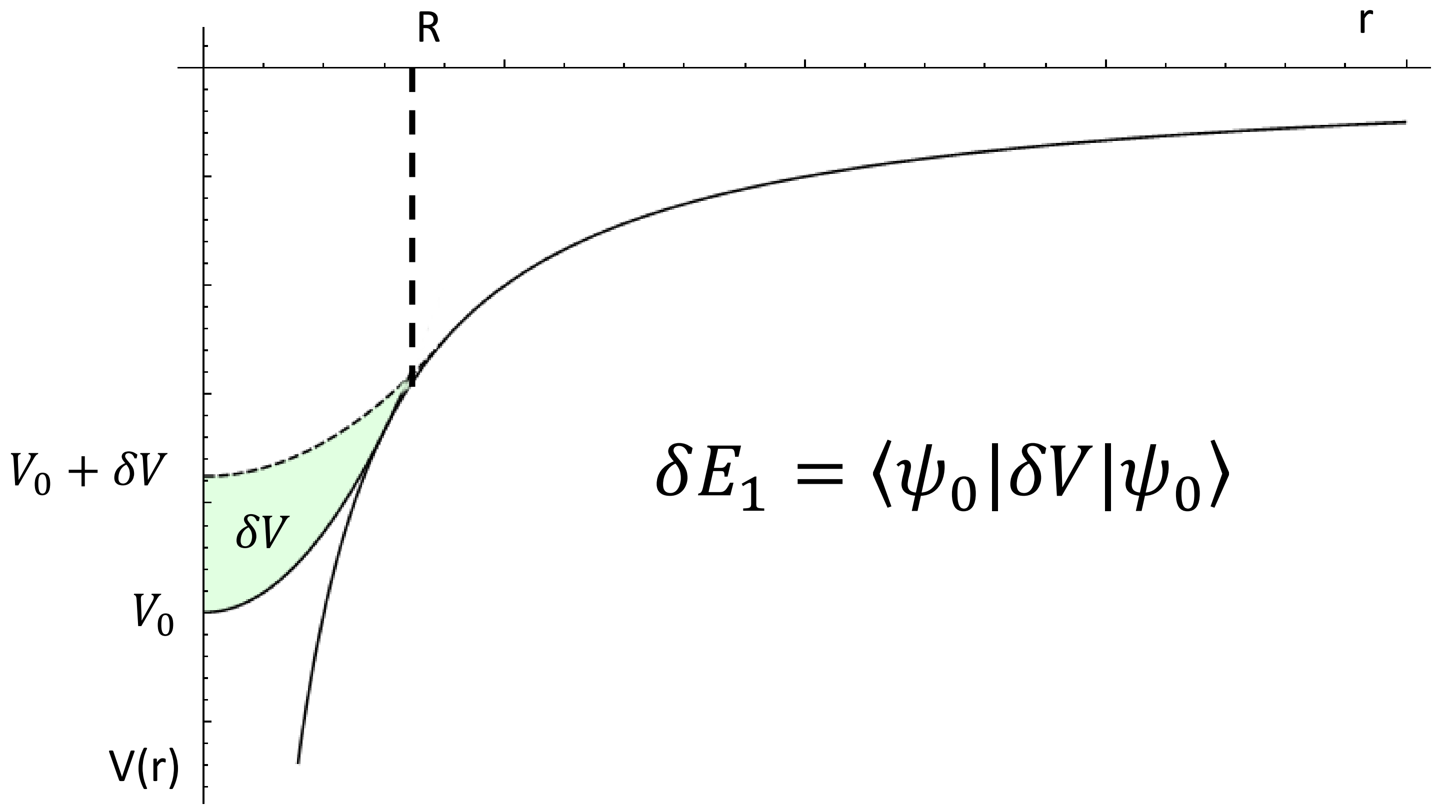
Field shift

single-electron mean-field approximation



Uniformly charged spherical nucleus

Perturbation



$$\delta E_1 = \langle \psi_0 | \delta V | \psi_0 \rangle$$

$$\delta E_\kappa = \frac{1}{z_i+1}\frac{12\,\kappa(\kappa-\gamma)}{(2|\kappa|+1)(2|\kappa|+3)[\Gamma(2\gamma+1)]^2} \times \left(\frac{2ZR}{a_B}\right)^{2\gamma}\sqrt{\frac{I_\kappa^3}{Ry}}\frac{\delta R}{R}$$

Z – nuclear charge

z_i – ion charge

κ – Dirac quantum number

$\kappa =$	-1	1	-2	2	-3	…
	$s_{1/2}$	$p_{1/2}$	$p_{3/2}$	$d_{3/2}$	$d_{5/2}$	…

$$\gamma = \sqrt{\kappa^2 - (\alpha Z)^2}$$

R – (equivalent) nuclear charge radius

$$\langle r^2 \rangle = \frac{3}{5}R^2$$

$\delta R = R_2 - R_1$

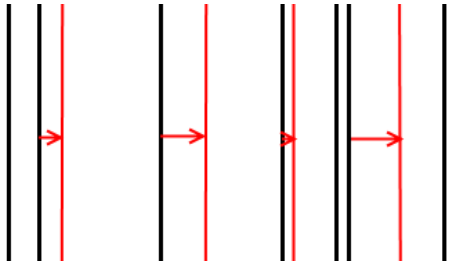
I_κ – ionization potential of the electron

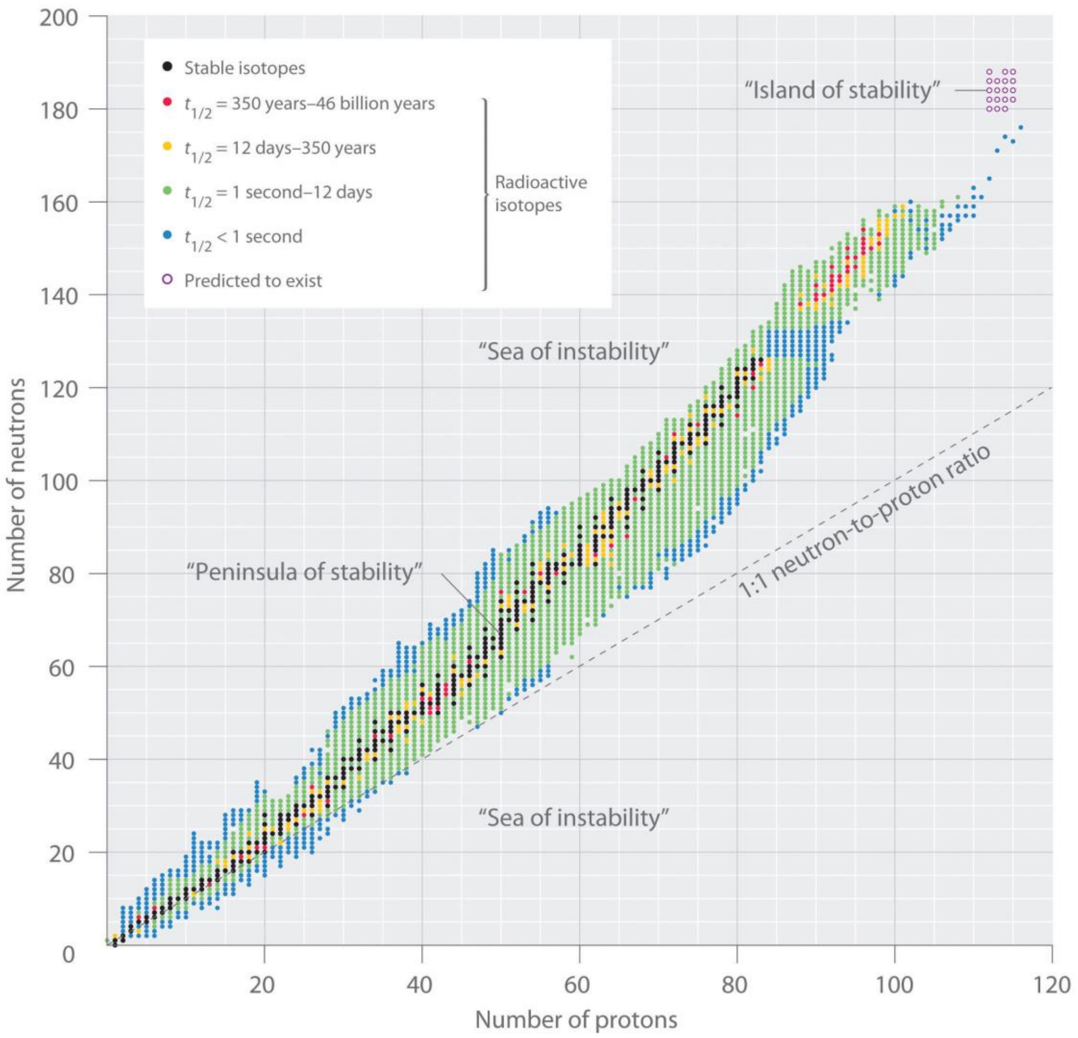
a_B – Bohr radius

Ry – Rydberg constant

Island of stability

- Isotopes with (theoretically predicted) magic neutron number N = 184 are not produced in laboratories.
- Possibility: find them in astrophysical data? [1]
- Calculate isotope shifts for N = 184 isotopes and add them to spectra of synthesized isotopes.





[1] V. A. Dzuba, V. V. Flambaum, and J. K. Webb, arXiv:1703.04250 (2017)

Figure source: B. Averill and P. Eldredge. *General Chemistry: Principles, Patterns, and Applications*. The Saylor Foundation, 2011.

Table I. Estimates of the isotope shift $\delta\nu$ for a given transition in superheavy atoms. A_1 is the atomic number of already synthesised reference isotope. $A_2 = Z + 184$ is the isotope of a given element belonging to the hypothetical island of stability with magic neutron number $N = 184$.

	Atom			Transition			$\delta\nu$ (cm^{-1})	$\delta\nu$ (GHz)
Symbol	Z	A_1	A_2					
Cf	98	251	282	$5f^{10}7s^2$	–	$5f^{10}7s7p$	-7.6	-229
Es	99	252	283	$5f^{11}7s^2$	–	$5f^{11}7s7p$	-8.3	-248
Fm	100	257	284	$5f^{12}7s^2$	–	$5f^{12}7s7p$	-7.8	-233
Md	101	258	285	$5f^{13}7s^2$	–	$5f^{13}7s7p$	-8.4	-253
No	102	259	286	$7s^2$	–	$7s7p$	-9.2	-275
Lr	103	266	287	$7s^27p$	–	$7s^28s$	0.78	23.3
Rf	104	263	288	$6d^27s^2$	–	$6d^27s7p$	-10	-300
Db	105	268	289	$6d^37s^2$	–	$6d^37s7p$	-9.1	-274
Sg	106	269	290	$6d^47s^2$	–	$6d^47s7p$	-9.9	-298
Bh	107	270	291	$6d^57s^2$	–	$6d^57s7p$	-11	-325
Hs	108	269	292	$6d^67s^2$	–	$6d^67s7p$	-13	-389
Mt	109	278	293	$6d^77s^2$	–	$6d^77s7p$	-9.2	-275
Ds	110	281	294	$6d^87s^2$	–	$6d^87s7p$	-8.7	-260
Rg	111	282	295	$6d^97s^2$	–	$6d^97s7p$	-9.5	-285
Cn	112	285	296	$6d^{10}7s^2$	–	$6d^{10}7s7p$	-8.8	-263
Nh	113	286	297	$7s^27p$	–	$7s^28s$	-0.35	-10.5
Fl	114	292	298	$7p^2$	–	$7p8s$	-0.64	-19.3

Non-linearity

$$\delta E_\kappa = \frac{1}{z_i + 1} \frac{12\,\kappa(\kappa - \gamma)}{(2|\kappa| + 1)(2|\kappa| + 3)[\Gamma(2\gamma + 1)]^2} \times \left(\frac{2ZR}{a_B}\right)^{2\gamma} \sqrt{\frac{I_\kappa^3}{Ry} \frac{\delta R}{R}}$$

Z – nuclear charge

z_i – ion charge

κ – Dirac quantum number

$\kappa =$	-1	1	-2	2	-3	…
	$s_{1/2}$	$p_{1/2}$	$p_{3/2}$	$d_{3/2}$	$d_{5/2}$	…

$$\gamma = \sqrt{\kappa^2 - (\alpha Z)^2}$$

R – (equivalent) nuclear charge radius

$$\langle r^2 \rangle = \frac{3}{5} R^2$$

$\delta R = R_2 - R_1$

I_κ – ionization potential of the electron

a_B – Bohr radius

Ry – Rydberg constant

Non-linearity

$$\Delta E_\kappa = \frac{1}{z_i+1}\frac{12\,\kappa(\kappa-\gamma)}{2\gamma(2|\kappa|+1)(2|\kappa|+3)[\Gamma(2\gamma+1)]^2}\times\left(\frac{2Z}{a_b}\right)^{2\gamma}\sqrt{\frac{I_\kappa^3}{Ry}}\,({R_2}^{2\gamma}-{R_1}^{2\gamma})$$

Z – nuclear charge

z_i – ion charge

κ – Dirac quantum number

$\kappa =$	-1	1	-2	2	-3	…
	$s_{1/2}$	$p_{1/2}$	$p_{3/2}$	$d_{3/2}$	$d_{5/2}$	…

$$\gamma = \sqrt{\kappa^2-(\alpha Z)^2}$$

R – (equivalent) nuclear charge radius

$$\langle r^2\rangle = \frac{3}{5}R^2$$

$$\delta R = R_2 - R_1$$

I_κ – ionization potential of the electron

a_B – Bohr radius

Ry – Rydberg constant

Estimates for the non-linearities

Ion						Pair of transitions	Non-linearity (Hz)
	Z	A	A_1	A_2	A_3		Higher waves
Ca^+	20	40	42	44	48	$3p^6 4s\ ^2S_{1/2} \to 3p^6 3d\ ^2D_{3/2}$ $3p^6 4s\ ^2S_{1/2} \to 3p^6 3d\ ^2D_{5/2}$	1.6×10^{-4}
Sr^+	38	84	86	87	88	$4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{3/2}$ $4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{5/2}$	-1.7×10^{-3}
Sr^+	38	84	86	88	90	$4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{3/2}$ $4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{5/2}$	-1.1×10^{-2}
Yb^+	70	168	170	172	176	$4f^{14} 6s\ ^2S_{1/2} \to 4f^{13} 6s^2\ ^2F^o_{7/2}$ $4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{3/2}$	-3.1
Yb^+	70	168	170	172	176	$4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{3/2}$ $4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{5/2}$	3.1
Hg^+	80	196	198	200	204	$5d^{10} 6s\ ^2S_{1/2} \to 5d^9 6s^2\ ^2D_{3/2}$ $5d^{10} 6s\ ^2S_{1/2} \to 5d^9 6s^2\ ^2D_{5/2}$	3.1

Higher-order SM contributions

- FS affected by higher waves in the transition ($p_{3/2}$, $d_{3/2}$, $d_{5/2}$ ...)
- Nuclear polarizability
- Many-body effects

Nuclear polarizability

$$V_\alpha = -\frac{1}{2}\frac{\alpha_P e^2}{r^4}$$

Migdal formula
for nuclear polarizability [1]:

$$\alpha_P = \frac{e^2 R^2 A}{40\, a_{sym}}$$

$a_{sym} \approx 23$ MeV

$$\delta E_\alpha = \int_{r_0}^{\infty} \rho_\kappa\,(r)\left(-\frac{1}{2}\frac{\alpha_P e^2}{r^4}\right) r^2 dr$$

$$r_0 = \begin{cases} R & ,|\kappa| = 1, \\ 0 & ,|\kappa| > 1. \end{cases}$$

[1] A.B. Migdal, JETP **15**, 81 (1945)

Estimates for the non-linearities

Ion						Pair of transitions	Non-linearity (Hz)	
	Z	A	A_1	A_2	A_3		Higher waves	+ polarizability
Ca^+	20	40	42	44	48	$3p^6 4s\ ^2S_{1/2} \to 3p^6 3d\ ^2D_{3/2}$ $3p^6 4s\ ^2S_{1/2} \to 3p^6 3d\ ^2D_{5/2}$	1.6×10^{-4}	-4.1×10^{-2}
Sr^+	38	84	86	87	88	$4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{3/2}$ $4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{5/2}$	-1.7×10^{-3}	1.7×10^{-1}
Sr^+	38	84	86	88	90	$4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{3/2}$ $4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{5/2}$	-1.1×10^{-2}	-2.6
Yb^+	70	168	170	172	176	$4f^{14} 6s\ ^2S_{1/2} \to 4f^{13} 6s^2\ ^2F^o_{7/2}$ $4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{3/2}$	-3.1	38
Yb^+	70	168	170	172	176	$4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{3/2}$ $4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{5/2}$	3.1	-18
Hg^+	80	196	198	200	204	$5d^{10} 6s\ ^2S_{1/2} \to 5d^9 6s^2\ ^2D_{3/2}$ $5d^{10} 6s\ ^2S_{1/2} \to 5d^9 6s^2\ ^2D_{5/2}$	3.1	-14

Higher-order SM contributions

- FS affected by higher waves in the transition ($p_{3/2}$, $d_{3/2}$, $d_{5/2}$ ...)
- Nuclear polarizability
- $\boxed{\text{Many-body effects}}$

Many-body corrections

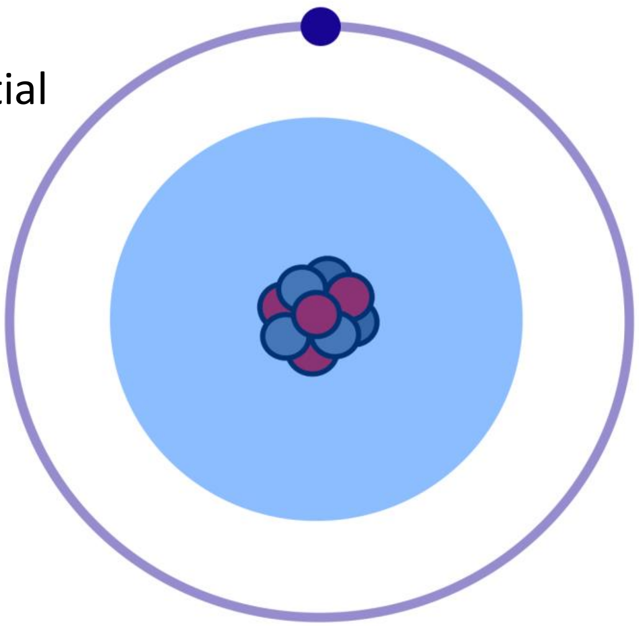
Rough estimation

$$\Delta\nu_{i,AA'} = K_i \mu_{AA'} + F_i \delta\langle r_{AA'}^{2\gamma_1}\rangle + G_i \delta\langle r_{AA'}^{2\gamma_2}\rangle + \cdots$$

- Reaction of the core electrons to the change of nuclear radius scales roughly as s-wave => **not much effect on the non-linearity**

$$\Delta\widetilde{E_\kappa} = \Delta E_\kappa - \lambda\, \Delta\varepsilon_s \left(\frac{I_\kappa}{I_s}\right)^{3/2}$$

I_κ – ionization potential of the electron



From the comparison with numerical calculation data [1]:

$\lambda = \frac{1}{2}$ for $\kappa \neq -1$

$\lambda \approx 0$ for $\kappa = -1$ (s-wave)

Here $\Delta\varepsilon_s = \Delta E_s$ or $\Delta\varepsilon_s = \Delta E_s + \Delta E_{s,\alpha}$.

[1] Berengut et al., arXiv: 1704.05068

Quadratic terms

$$\Delta\widetilde{E_\kappa} = \Delta E_\kappa - \lambda\, \Delta\varepsilon_s \left(\frac{I_\kappa}{I_s}\right)^{3/2}$$

$$\Delta\widetilde{\widetilde{E_\kappa}} = \Delta\widetilde{E_\kappa} \pm \frac{\left(\Delta\widetilde{E_\kappa}\right)^2}{I_\kappa}$$

Estimates for the non-linearities

Ion						Pair of transitions	Non-linearity (Hz)		
	Z	A	A_1	A_2	A_3		HW & Polariz.	HW & Polariz. & MB	$\pm$ Quadratic
Ca^+	20	40	42	44	48	$3p^6 4s\ ^2S_{1/2} \rightarrow 3p^6 3d\ ^2D_{3/2}$ $3p^6 4s\ ^2S_{1/2} \rightarrow 3p^6 3d\ ^2D_{5/2}$	-4.1×10^{-2}	-4.1×10^{-2}	$\pm 1.4 \times 10^{-3}$
Sr^+	38	84	86	87	88	$4p^6 5s\ ^2S_{1/2} \rightarrow 4p^6 4d\ ^2D_{3/2}$ $4p^6 5s\ ^2S_{1/2} \rightarrow 4p^6 4d\ ^2D_{5/2}$	1.7×10^{-1}	1.7×10^{-1}	$\mp 3.7 \times 10^{-2}$
Yb^+	70	168	170	172	176	$4f^{14} 6s\ ^2S_{1/2} \rightarrow 4f^{13} 6s^2\ ^2F^o_{7/2}$ $4f^{14} 6s\ ^2S_{1/2} \rightarrow 4f^{14} 5d\ ^2D_{3/2}$	38	39	± 12130
Yb^+	70	168	170	172	176	$4f^{14} 6s\ ^2S_{1/2} \rightarrow 4f^{14} 5d\ ^2D_{3/2}$ $4f^{14} 6s\ ^2S_{1/2} \rightarrow 4f^{14} 5d\ ^2D_{5/2}$	-18	-18	386
Hg^+	80	196	198	200	204	$5d^{10} 6s\ ^2S_{1/2} \rightarrow 5d^9 6s^2\ ^2D_{3/2}$ $5d^{10} 6s\ ^2S_{1/2} \rightarrow 5d^9 6s^2\ ^2D_{5/2}$	-14	-13	± 2382

Preliminary

New Physics in King plot?

$$V_\varphi = -q_e q_n N \frac{e^{-kr}}{r}$$

$$k = \frac{m_\varphi c}{\hbar}, \quad \alpha_{NP} \equiv \frac{q_e q_n}{\hbar c}$$

$$\delta E_{NP} = -q_e q_n \Delta N \int_0^\infty \rho_\kappa(r) \frac{e^{-kr}}{r} r^2 dr$$

What new particle's α_{NP} could produce the same non-linearity, as we have found for SM effects?

Estimates for the new interactions

Ion	Z	A	A₁	A₂	A₃	Pair of transitions	Non-linearity (Hz)	$\frac{\alpha_{NP}}{\alpha}$ $m_\phi \to 0$	$m_\phi = 10^5$ eV	$m_\phi = 10^6$ eV	$m_\phi = 10^7$ eV
Ca^+	20	40	42	44	48	$3p^6 4s\ ^2S_{1/2} \to 3p^6 3d\ ^2D_{3/2}$ $3p^6 4s\ ^2S_{1/2} \to 3p^6 3d\ ^2D_{5/2}$	-1.5	8.2×10^{-12}	2.1×10^{-9}	-1.4×10^{-8}	-1.1×10^{-6}
Sr^+	38	84	86	87	88	$4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{3/2}$ $4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{5/2}$	0.7	7.0×10^{-13}	5.7×10^{-11}	-7.7×10^{-10}	-3.9×10^{-8}
Yb^+	70	168	170	172	176	$4f^{14} 6s\ ^2S_{1/2} \to 4f^{13} 6s^2\ ^2F^o_{7/2}$ $4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{3/2}$	12190	-2.5×10^{-11}	2.4×10^{-9}	1.3×10^{-8}	3.2×10^{-7}
						$4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{3/2}$ $4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{5/2}$	-406	2.7×10^{-11}	2.2×10^{-9}	-1.7×10^{-8}	-3.9×10^{-7}
Hg^+	80	196	198	200	204	$5d^{10} 6s\ ^2S_{1/2} \to 5d^9 6s^2\ ^2D_{3/2}$ $5d^{10} 6s\ ^2S_{1/2} \to 5d^9 6s^2\ ^2D_{5/2}$	-2395	-1.8×10^{-10}	6.6×10^{-8}	-5.5×10^{-8}	-1.0×10^{-6}

Super-Preliminary

To be continued ...

Backup slides

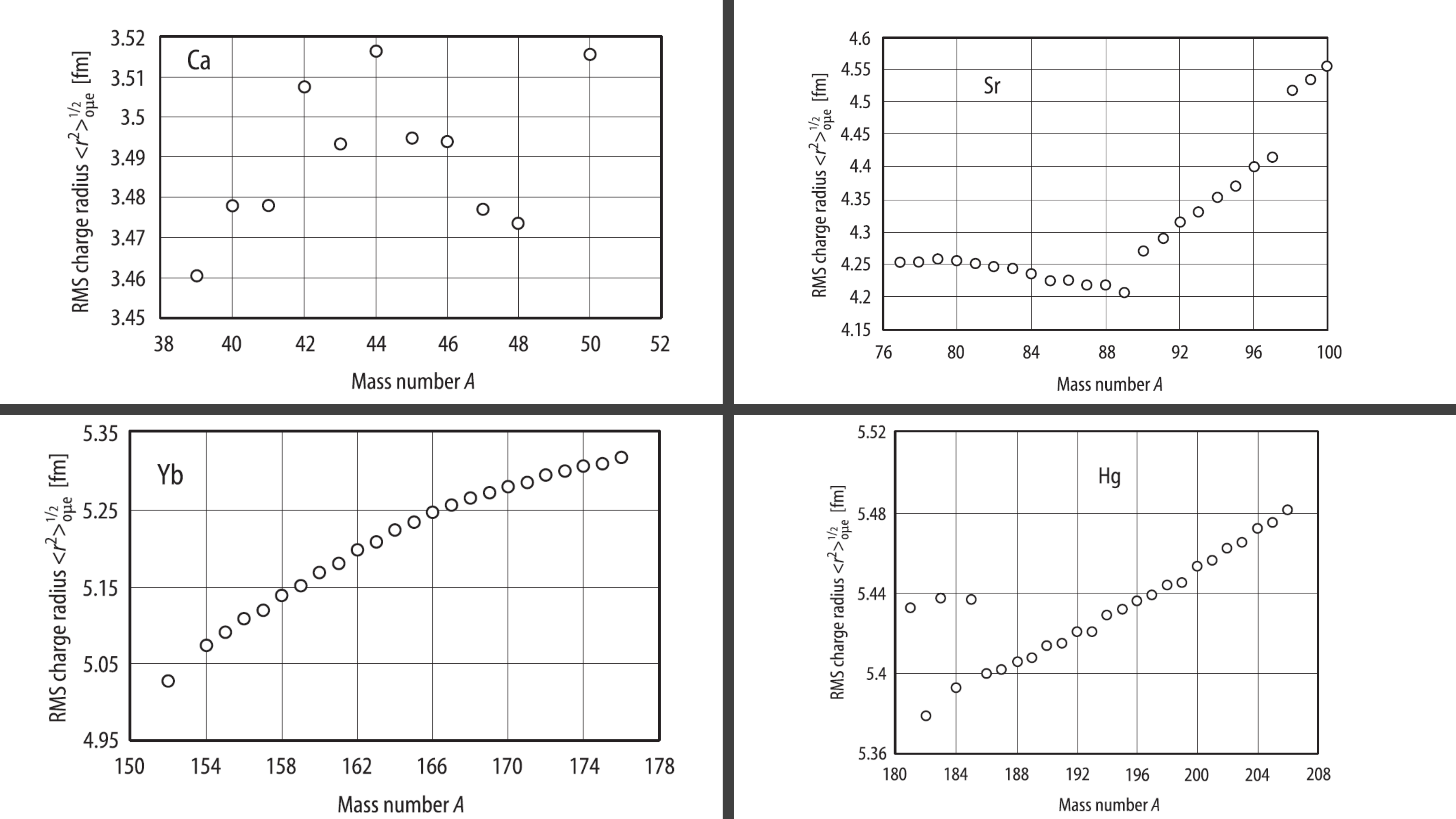
Element	Transition	IS, experiment (MHz)	IS, theory (MHz)
Ca (A=46-48)	3p6 4s2 – 3p6 4s 4p	-25.3 ± 1.0	-31
Yb (A=174-176)	4f14 6s2 – 4f14 6s 6p	993 ± 250	1217
Hg (A=202-204)	5d10 6s2 – 6d10 6s 6p	5238 ± 11	4939

CI+MBPT

No (A=259-286)	7s2 - 7s 7p	-7.2 cm^-1	-9.2 cm^-1

[1] Fricke, G., Heilig, K. "Nuclear charge radii" (2004).

[2] V. A. Dzuba, V. V. Flambaum, and J. K. Webb, arXiv:1703.04250 (2017)


Ca
RMS charge radius $<r^2>^{1/2}_{o\mu e}$ [fm]
3.45
3.46
3.47
3.48
3.49
3.5
3.51
3.52
38
40
42
44
46
48
50
52
Mass number A
Sr
RMS charge radius $<r^2>^{1/2}_{o\mu e}$ [fm]
4.15
4.2
4.25
4.3
4.35
4.4
4.45
4.5
4.55
4.6
76
80
84
88
92
96
100
Mass number A
Yb
RMS charge radius $<r^2>^{1/2}_{o\mu e}$ [fm]
4.95
5.05
5.15
5.25
5.35
150
154
158
162
166
170
174
178
Mass number A
Hg
RMS charge radius $<r^2>^{1/2}_{o\mu e}$ [fm]
5.36
5.4
5.44
5.48
5.52
180
184
188
192
196
200
204
208
Mass number A

Mass shift

Energy difference between the infinite and finite mass isotopes:

$$\Delta E_{\infty,M} = \boxed{\frac{\langle \sum_i p_i^2 \rangle}{2(M+m)}} + \boxed{\frac{\langle \sum_{i>j} \mathbf{p}_i \mathbf{p}_j \rangle}{(M+m)}}$$

NMS: change of the reduced mass
(single-electron effect)

SMS: electron correlations
(many-electron effect)

(M – nuclear mass, m – electron mass, p – electron momenta)

Mass shift

Energy difference between the infinite and finite mass isotopes:

$$\Delta E_{\infty,M} = \frac{K}{(M+m)}$$

$$\Delta E_{M_1,M_2} = \frac{K(M_1 - M_2)}{(M_1+m)(M_2+m)} \approx K\mu \qquad , m \ll M$$

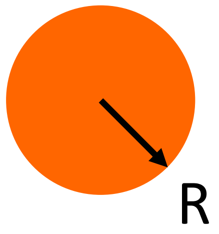
$$\mu = \frac{1}{M_1} - \frac{1}{M_2}$$

(M – nuclear mass, m – electron mass, p – electron momenta)

Field shift

Mean square charge radius:

$$\langle r^2 \rangle = \frac{\int_0^\infty r^4 \rho(r) dr}{\int_0^\infty r^2 \rho(r) dr}$$



$$\langle r^2 \rangle = \frac{\int_0^R r^4 dr}{\int_0^R r^2 dr} = \frac{3}{5} R^2$$