We investigate the possibility of realizing supersolid quantum phases in bosonic Rydberg-excited quantum lattice gases in the presence of non-unitary processes, by simulating the dynamical evolution starting from initial preparation in non-dissipative equilibrium states. Within Gutzwiller theory, we first analyze the many-body ground-state of a bosonic Rydberg-excited quantum gas in a two dimensional optical lattice for variable atomic hopping rates and Rabi detunings. Furthermore, we perform time evolution of different supersolid phases using the Lindblad-master equation. With the inclusion of two different non-unitary processes, namely spontaneous decay from a Rydberg state to the ground state and dephasing of the addressed Rydberg state, we study the effect of non-unitary processes on those quantum phases and observe long-lived states in the presence of decay and dephasing. We find that long-lived supersolid quantum phases are observable within a range of realistic decay and dephasing rates, while high rates cause any initial configuration to homogenize quickly, preventing possible supersolid formation.