Quantum mechanics of fluids in solids

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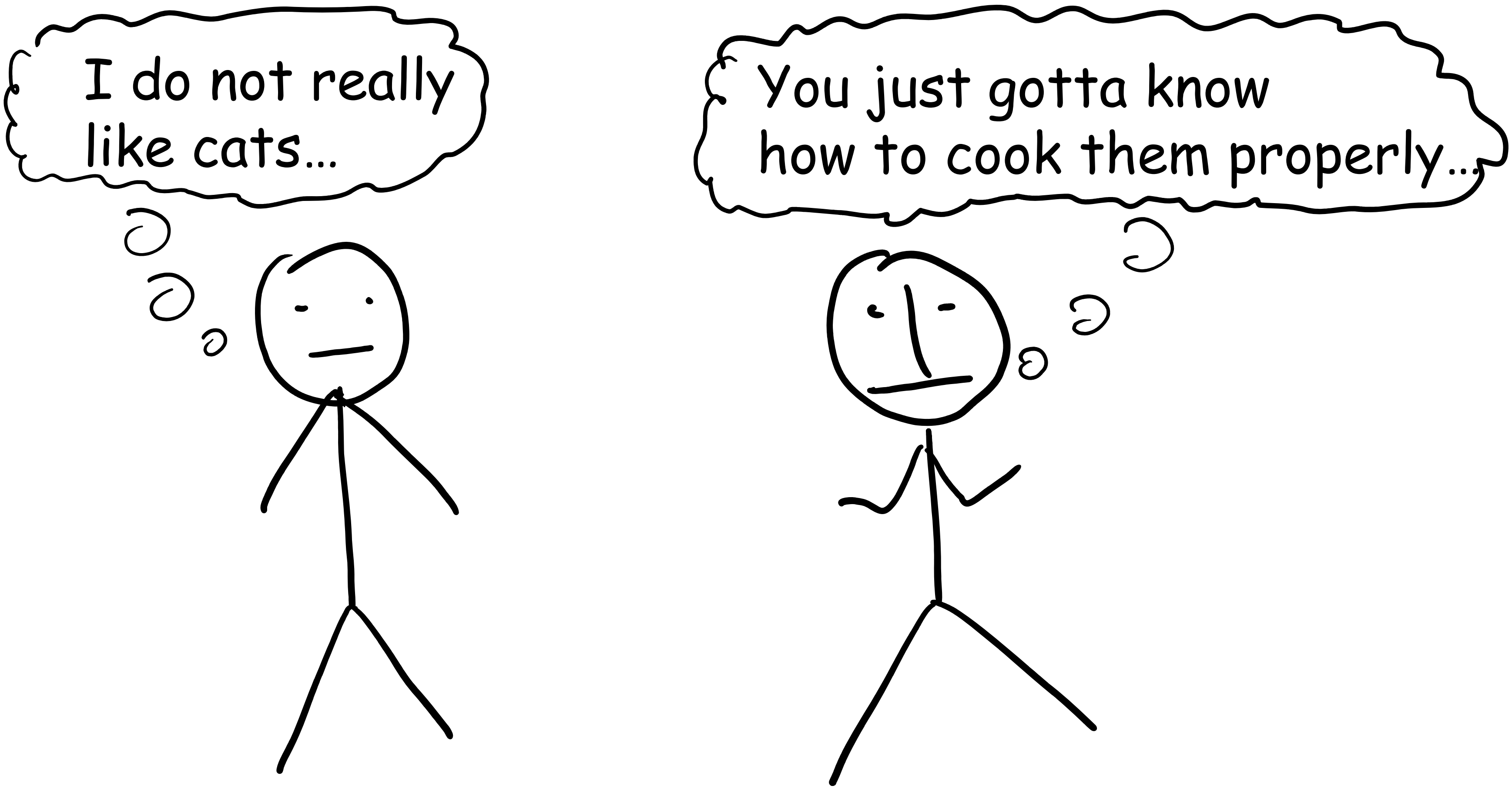
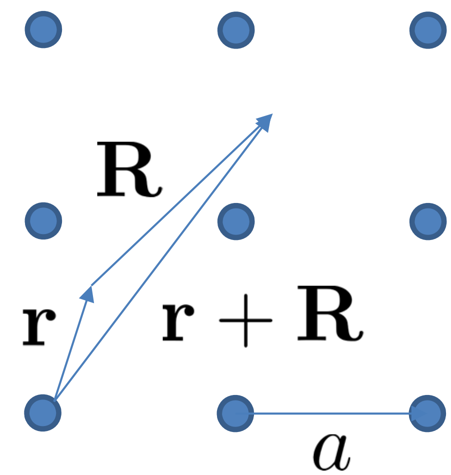
Negativity toward the lattice be like…



Band theory of solids



Lattice translation symmetry:

$$[H, T(\mathbf{R})] = 0, \;\; T(\mathbf{R})|\Psi\rangle = e^{i\mathbf{kR}}|\Psi\rangle$$

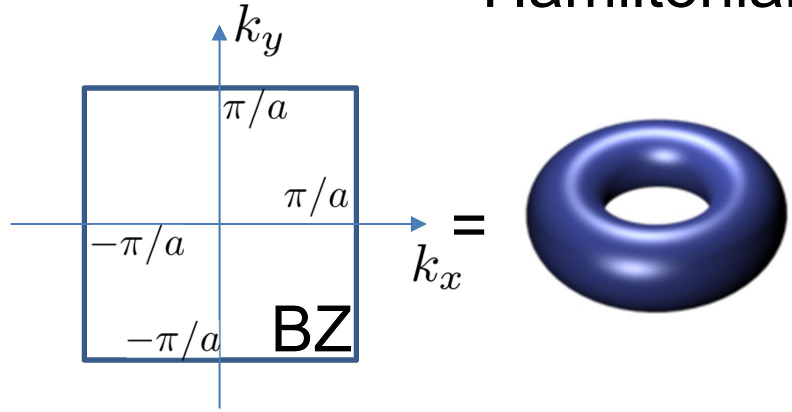
Bloch theorem: $\Psi_{n\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{kr}} u_{n\mathbf{k}}(\mathbf{r})$

Schrödinger equation:

$$H|\Psi\rangle = E_{n\mathbf{k}}|\Psi\rangle$$

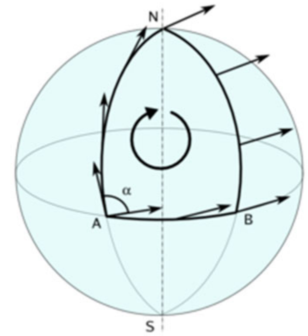
$$H_{\mathbf{k}}|u_{n\mathbf{k}}\rangle = E_{n\mathbf{k}}|u_{n\mathbf{k}}\rangle, \quad H_{\mathbf{k}} = e^{-i\mathbf{kr}} H e^{i\mathbf{kr}} \text{ — }$$ Bloch Hamiltonian

$\mathbf{k}$ - “quasimomentum”.
Physically distinct ones belong to the Brillouin zone:



Two types of geometry in metals

a) "Lifshitz-Azbel-Kaganov" geometry:
geometry of iso-energetic surfaces,
$E_{n\mathbf{k}} = \mathrm{const}$, led to "Fermiology"



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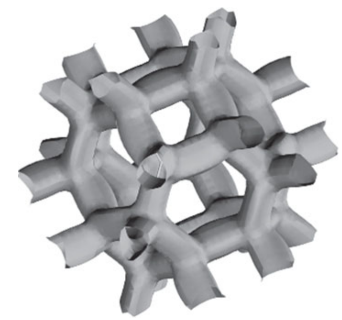


$$d\alpha = A_\theta d\theta + A_\varphi d\varphi$$

Fermi surface of Pb

b) "Pancharatnam-Berry" geometry:
geometry of wave functions

$$\gamma_C = \int_C d\mathbf{k} \cdot \mathbf{A}_\mathbf{k}$$

$\mathbf{A}_\mathbf{k} = i\langle u_\mathbf{k} | \nabla_\mathbf{k} | u_\mathbf{k} \rangle$ - Berry connection

$$e^{i\gamma_C} |u_{\mathbf{k}_f}\rangle$$

$$|u_{\mathbf{k}_i}\rangle \quad C$$

Examples of geometric phases abound in many areas of physics. Many familiar problems that we do not ordinarily associate with geometric phases may be phrased in terms of them. Often, the result is a clearer understanding of the structure of the problem, and an elegant expression of its solution.

M. V. Berry,
in Shapere&Wilczek, "Geometric phases in physics"

Secret: one can do anything *without* using the geometric phase stuff

Classical mechanics of electrons in solids

Promote the band energy to the classical Hamiltonian:

$$H(\boldsymbol{r},\boldsymbol{k}) = E_{\boldsymbol{k}-e\boldsymbol{A}} + e\phi(\boldsymbol{r})$$

"Peierls substitution"

Write down the equations of motion:

$$\dot{\boldsymbol{r}} = \partial_{\boldsymbol{k}} E_{\boldsymbol{k}},$$

$$\dot{\boldsymbol{k}} = -\partial_{\boldsymbol{r}} E_{\boldsymbol{k}} + e\partial_{\boldsymbol{k}} E_{\boldsymbol{k}} \times \boldsymbol{B}.$$

Then perhaps solve the Boltzmann equation:

$$\partial_t f + \dot{\boldsymbol{r}}\boldsymbol{\nabla} f + \dot{\boldsymbol{k}}\partial_{\boldsymbol{k}} f = \hat{I}_{st}$$

We would like to describe deviations from this picture, i.e. the departure from the classical point of view.

Berry phases in adiabatic evolution

Define the Bloch Hamiltonian: $H_{\mathbf{k}}|u_{n\mathbf{k}}\rangle = E_{n\mathbf{k}}|u_{n\mathbf{k}}\rangle$

$$H_{\boldsymbol{k}} = e^{-i\boldsymbol{k}\boldsymbol{r}}\left(\frac{\hat{\boldsymbol{p}}^2}{2m} + U(\boldsymbol{r})\right)e^{i\boldsymbol{k}\boldsymbol{r}} = \frac{(\hat{\boldsymbol{p}}+\boldsymbol{k})^2}{2m} + U(\boldsymbol{r}).$$

For $\boldsymbol{k} \to \boldsymbol{k}(t)$, define instantaneous eigensystem:

$$H_{\mathbf{k}}|u_{n\mathbf{k}}\rangle = E_{n\mathbf{k}}|u_{n\mathbf{k}}\rangle$$

Project the dynamics onto a given band, $\langle u_{n\boldsymbol{k}(t)}|(\ldots S.E.\ldots.)$:

$$i\partial_t|\psi\rangle = H_{\boldsymbol{k}(t)}|\psi\rangle, \;\; |\psi\rangle = c(t)e^{-i\int^t E_{\boldsymbol{k}(t')}dt'}|u_{n\boldsymbol{k}}\rangle$$

In this case, only a phase can be accumulated:

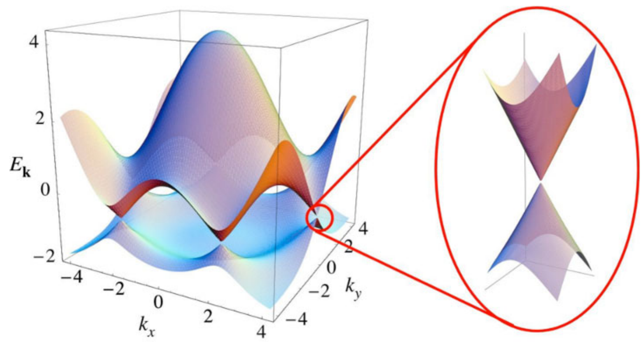
$$\dot{c}(t) = -\langle u_{\boldsymbol{k}(t)}|\partial_t|u_{\boldsymbol{k}(t)}\rangle c(t),$$

$$c(t) = c(0)e^{i\gamma_B},$$

$$\boxed{\gamma_C = \int_C d\mathbf{k}\cdot\mathbf{A}_{\mathbf{k}}}$$

$$\boxed{\mathbf{A}_{\mathbf{k}} = i\langle u_{\mathbf{k}}|\nabla_{\mathbf{k}}|u_{\mathbf{k}}\rangle}$$

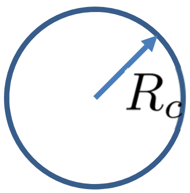
Example: Landau levels in graphene



$$H_{\boldsymbol{k}} = v\boldsymbol{\sigma k} = v(\sigma_x k_x + \sigma_y k_y)$$

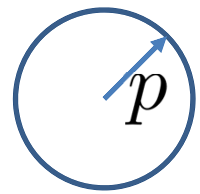
$$|u_{\boldsymbol{k}}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\phi_{\boldsymbol{k}}} \end{pmatrix}$$

Trajectory in r-space:



$$R_c = \frac{pc}{eB} \qquad d\boldsymbol{p} = \frac{e}{c} d\boldsymbol{r} \times \boldsymbol{B}$$

Trajectory in p-space:



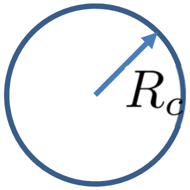
Berry connection and Berry curvature:

$$A_\phi = \frac{i}{k}\langle u_{\boldsymbol{k}} | \partial_\phi u_{\boldsymbol{k}}\rangle = -\frac{1}{2k}$$

$$\gamma_C = -\int_0^{2\pi} d\phi k A_\phi = \pi$$

Example: Landau levels in graphene

Trajectory in r-space:



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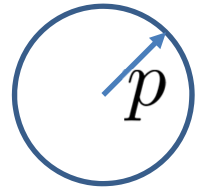


Semiclassical (Lifshitz-Onsager) quantization:

$$\gamma_{kin} + \gamma_{A-B} + \gamma_C = 2\pi(n + 1/2)$$

$$\frac{1}{\hbar}\oint \boldsymbol{p} d\boldsymbol{r} = 2\frac{c}{\hbar eB}\pi p^2 \qquad \frac{\Phi}{\Phi_0} = -\frac{c}{\hbar eB}\pi p^2 \qquad \pi$$

$$\boxed{\frac{c}{\hbar eB}\pi\frac{\varepsilon^2}{v^2} = 2\pi n \Rightarrow \varepsilon = \pm\sqrt{2\hbar v^2 eBn/c}}$$

Note a Landau level
at n=0!

Berry curvature

"Magnetic field" that corresponds to the Berry connection:

$$\boldsymbol{\Omega}_{n\boldsymbol{k}} = \boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{A}_{\boldsymbol{k}} = i\langle\partial_{\boldsymbol{k}} u_{n\boldsymbol{k}}| \times |\partial_{\boldsymbol{k}} u_{n\boldsymbol{k}}\rangle$$

By Stokes theorem,

$$\gamma_C = \oint d\boldsymbol{S}\boldsymbol{\Omega}_{\boldsymbol{k}}$$

Symmetry properties:

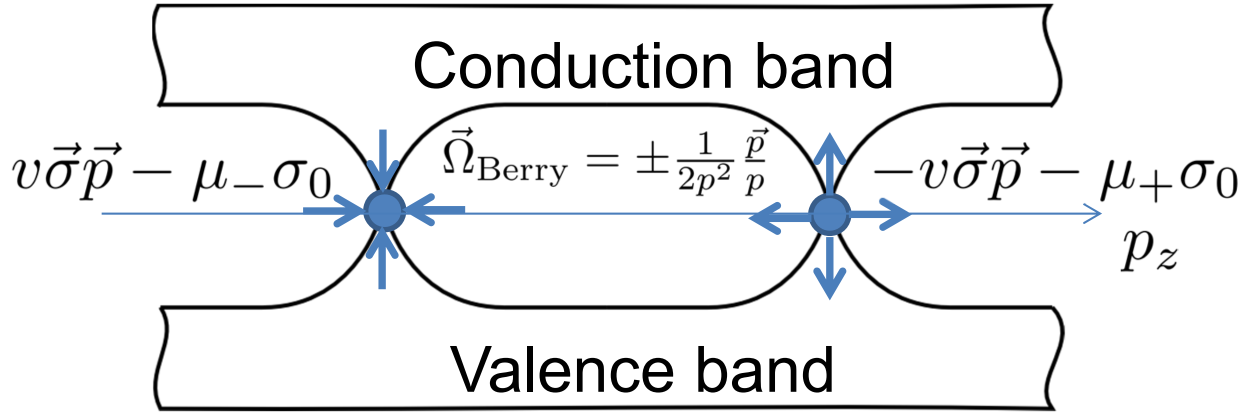
I: $\boldsymbol{\Omega}(\boldsymbol{k}) = \boldsymbol{\Omega}(-\boldsymbol{k})$

TR: $\boldsymbol{\Omega}(\boldsymbol{k}) = -\boldsymbol{\Omega}(-\boldsymbol{k})$

Either I or TR have to be broken to have non-zero Berry curvature

Example: Berry curvature in a Weyl semimetal

3D "graphene",
"gapless topo phase":



First materials:

Dirac metals: Cd_3As_2 , Na_3Bi

Liang et al., Nat. Mat. (2015)

$ZnTe_5$:

Li et al., Nature Physics 12, 550-554 (2016)

Transition metal pnictides (TaAs, TaP, etc):

S.-Y. Xu et al., Science 349, 613 (2015).

B. Q. Lv et al., Phys. Rev. X 5, 031013 (2015).

Reviews: Vafek&Vishwanath, Ann. Rev. 2014, Yan&Felser, Ann. Rev. 2017

Example: Berry curvature in a Weyl semimetal

Simplest low-energy (Weyl) Hamiltonian: $H_{\boldsymbol{k}} = v\boldsymbol{\sigma}\boldsymbol{k}$

$$\boldsymbol{\Omega}_{n\boldsymbol{k}} = i\langle\partial_{\boldsymbol{k}} u_{n\boldsymbol{k}}| \times |\partial_{\boldsymbol{k}} u_{n\boldsymbol{k}}\rangle$$

$$= i\sum_{m\neq n} \langle\partial_{\boldsymbol{k}} u_{n\boldsymbol{k}}|u_{m\boldsymbol{k}}\rangle \times \langle u_{m\boldsymbol{k}}|\partial_{\boldsymbol{k}} u_{n\boldsymbol{k}}\rangle$$

Use $\langle u_m|\partial_{\boldsymbol{k}} u_n\rangle = \frac{\langle u_m|\partial_{\boldsymbol{k}} H_{\boldsymbol{k}}|u_n\rangle}{\varepsilon_n - \varepsilon_m}$ to obtain

$$\boxed{\boldsymbol{\Omega}_{n\boldsymbol{k}} = \sum\nolimits_m' \frac{\langle u_n|\partial_{\boldsymbol{k}} H_{\boldsymbol{k}}|u_m\rangle \times \langle u_m|\partial_{\boldsymbol{k}} H_{\boldsymbol{k}}|u_n\rangle}{(\varepsilon_n - \varepsilon_m)^2}}$$

i) used for practical calculations,
ii) shows that we are dealing with a case of interband coherence

For a Weyl point,

$$\boxed{\boldsymbol{\Omega}_{n\boldsymbol{k}}^{Weyl} = -\frac{1}{2}\frac{1}{k^2}\hat{\boldsymbol{k}}, \ \mathrm{div}_{\boldsymbol{k}}\boldsymbol{\Omega}_{n\boldsymbol{k}} = -2\pi Q_W \delta(\boldsymbol{k} - K_W).}$$

Semiclassical dynamics of band electrons

Position operator in the band representation

Wave function in the band representation:

$$|\psi\rangle = \sum_{n\boldsymbol{k}} c_{n\boldsymbol{k}} e^{i\boldsymbol{kr}} |u_{n\boldsymbol{k}}\rangle, \;\; \hat{\boldsymbol{r}} c_{n\boldsymbol{k}} = ?$$

Minimal derivation:

$$\hat{\boldsymbol{r}}|\psi\rangle = \sum_{n\boldsymbol{k}} c_{n\boldsymbol{k}} \frac{1}{i}(\partial_{\boldsymbol{k}} e^{i\boldsymbol{kr}})|u_{n\boldsymbol{k}}\rangle = \sum_{n\boldsymbol{k}} e^{i\boldsymbol{kr}} i\partial_{\boldsymbol{k}}(c_{n\boldsymbol{k}}|u_{n\boldsymbol{k}}\rangle)$$

$$= \sum_{n\boldsymbol{k}} e^{i\boldsymbol{kr}} (i\partial_{\boldsymbol{k}} c_{n\boldsymbol{k}}|u_{n\boldsymbol{k}} + c_{n\boldsymbol{k}} i\partial_{\boldsymbol{k}}|u_{n\boldsymbol{k}}\rangle)$$

$$\to \sum_{n\boldsymbol{k}} e^{i\boldsymbol{kr}} |u_{n\boldsymbol{k}}\rangle (i\partial_{\boldsymbol{k}} + i\langle u_{n\boldsymbol{k}}|\partial_{\boldsymbol{k}}|u_{n\boldsymbol{k}}\rangle) c_{n\boldsymbol{k}} \equiv \sum_{n\boldsymbol{k}} e^{i\boldsymbol{kr}} |u_{n\boldsymbol{k}}\rangle (\hat{\boldsymbol{r}} c_{n\boldsymbol{k}})$$

Position operator projected onto a band in a crystal:

$$\hat{\mathbf{r}} = i\nabla_{\mathbf{k}} + \mathbf{A}_{\mathbf{k}}, \;\; \mathbf{A}_{\mathbf{k}} = i\langle u_{n\mathbf{k}}|\partial_{\mathbf{k}} u_{n\mathbf{k}}\rangle$$

Lattice coordinate   Coordinate within the unit cell

Semiclassical motional in external fields

Proceed by comparison:

$$\hat{\mathbf{p}} = \frac{1}{i}\nabla_{\mathbf{r}} - e\boldsymbol{A}_{\mathbf{r}}$$

$$\hat{\mathbf{r}} = i\nabla_{\mathbf{p}} + \mathbf{A}_{\mathbf{p}}$$ - looks like a vector potential in momentum space

Motion in ***external*** fields is semiclassical:

$$\dot{\mathbf{p}} = -e\frac{\partial\phi}{\partial\mathbf{r}} + e\dot{\mathbf{r}}\times\mathbf{B},\ \mathbf{B} = \nabla_{\mathbf{r}}\times\boldsymbol{A}_{\mathbf{r}}$$

$$\dot{\mathbf{r}} = \frac{\partial\epsilon_{n\mathbf{p}}}{\partial\mathbf{p}} - \dot{\mathbf{p}}\times\boldsymbol{\Omega}_{n\mathbf{p}},\ \boldsymbol{\Omega}_{n\mathbf{p}} = \nabla_{\mathbf{p}}\times\mathbf{A}_{\mathbf{p}}$$

$$\boldsymbol{\Omega}_{n\mathbf{p}} = i\langle\partial_{\mathbf{p}}u_{n\mathbf{p}}|\times|\partial_{\mathbf{p}}u_{n\mathbf{p}}\rangle$$

Application: anomalous Hall effect (B=0)

$$\begin{aligned}\dot{\mathbf{p}} &= e\boldsymbol{E} + e\dot{\mathbf{r}}\times\mathbf{B} \\ \dot{\mathbf{r}} &= \frac{\partial\epsilon_{n\mathbf{p}}}{\partial\mathbf{p}} - \dot{\mathbf{p}}\times\boldsymbol{\Omega}_{n\mathbf{p}}\end{aligned} \quad \xrightarrow{B=0} \quad \begin{aligned}\dot{\mathbf{p}} &= e\mathbf{E} \\ \dot{\mathbf{r}} &= \frac{\partial\epsilon_{n\mathbf{p}}}{\partial\mathbf{p}} - e\mathbf{E}\times\boldsymbol{\Omega}_{n\mathbf{p}}\end{aligned}$$

$$\boldsymbol{j}^{\mathrm{AHE}} = e^2\int_{\mathbf{p}}\boldsymbol{\Omega}_{n\mathbf{p}}\times\boldsymbol{E}f_{n\mathbf{p}},\ \sigma_{ab}^{\mathrm{AHE}} = -e^2\epsilon_{abc}\int_{\mathbf{p}}\boldsymbol{\Omega}^c_{n\mathbf{p}}f_{n\mathbf{p}}.$$

Historical time scales:
discovery - E. Hall in 1881,
relation to the spin-orbit coupling - 1954 by Karplus&Luttinger,
relation to band geometry - 1982 by TKNN

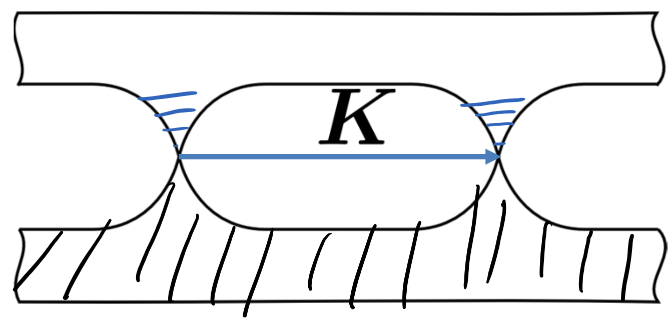
Compare to superconductivity&BCS: 1911-1957

Application: anomalous Hall effect (B=0)

$$\begin{aligned}\dot{\mathbf{p}} &= e\boldsymbol{E} + e\dot{\mathbf{r}}\times\mathbf{B}\\ \dot{\mathbf{r}} &= \frac{\partial\epsilon_{n\mathbf{p}}}{\partial\mathbf{p}} - \dot{\mathbf{p}}\times\boldsymbol{\Omega}_{n\mathbf{p}}\end{aligned} \quad \xrightarrow{B=0} \quad \begin{aligned}\dot{\mathbf{p}} &= e\mathbf{E}\\ \dot{\mathbf{r}} &= \frac{\partial\epsilon_{n\mathbf{p}}}{\partial\mathbf{p}} - e\mathbf{E}\times\boldsymbol{\Omega}_{n\mathbf{p}}\end{aligned}$$

$$\boldsymbol{j}^{\mathrm{AHE}} = e^2\int_{\mathbf{p}}\boldsymbol{\Omega}_{n\mathbf{p}}\times\boldsymbol{E}f_{n\mathbf{p}},\quad \sigma^{\mathrm{AHE}}_{ab} = -e^2\epsilon_{abc}\int_{\mathbf{p}}\boldsymbol{\Omega}^c_{n\mathbf{p}}f_{n\mathbf{p}}.$$

For a Weyl semimetal:



$$\int_{\boldsymbol{k}}\Omega^a_{\boldsymbol{k}} = \int_{\boldsymbol{k}}\Omega^b_{\boldsymbol{k}}\delta_{ab} = \int_{\boldsymbol{k}}\Omega^b_{\boldsymbol{k}}\partial_{k_b}k_a \overset{?}{=} -\int_{\boldsymbol{k}}k_a(\partial_{\boldsymbol{k}}\cdot\boldsymbol{\Omega}_{\boldsymbol{k}})$$

$$\sigma^{\mathrm{AHE}}_{ab} = \frac{e^2}{2\pi h}\epsilon_{abc}K_c$$

"almost quantized" 3D Hall effect.
K is defined up to a reciprocal lattice vector. (Haldane, PRL 2004)

Motion in magnetic field

$$
\begin{aligned}
\dot{\mathbf{p}} &= e\boldsymbol{E} + e\dot{\mathbf{r}} \times \mathbf{B} \\
\dot{\mathbf{r}} &= \mathbf{v_p} - \dot{\mathbf{p}} \times \boldsymbol{\Omega}_{n\mathbf{p}}
\end{aligned}
$$

Chiral anomaly

$$
\begin{aligned}
\dot{\mathbf{p}} &= \frac{1}{D_{\boldsymbol{B}}}\left(e\boldsymbol{E} + e\mathbf{v_p} \times \mathbf{B} - e^2(\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega}_{\mathbf{p}}\right) \\
\dot{\mathbf{r}} &= \frac{1}{D_{\boldsymbol{B}}}\left(\mathbf{v_p} - e\boldsymbol{E} \times \boldsymbol{\Omega}_{n\mathbf{p}} - e(\mathbf{v_p} \cdot \boldsymbol{\Omega}_{\mathbf{p}})\boldsymbol{B}\right)
\end{aligned}
$$

AHE

Chiral magnetic effect

$$
D_{\boldsymbol{B}} = 1 - e\boldsymbol{B}\boldsymbol{\Omega}_{\boldsymbol{p}}
$$

Chiral magnetic effect

$$\boldsymbol{j} = e \int_{\boldsymbol{p}} D_{\boldsymbol{B}} \dot{\boldsymbol{r}}$$

$$\begin{aligned}
\dot{\mathbf{p}} &= \frac{1}{D_{\boldsymbol{B}}}\left(e\boldsymbol{E} + e\mathbf{v}_{\mathbf{p}} \times \mathbf{B} - e^2(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{\Omega}_{\mathbf{p}}\right) \\
\dot{\mathbf{r}} &= \frac{1}{D_{\boldsymbol{B}}}\left(\mathbf{v}_{\mathbf{p}} - e\boldsymbol{E} \times \boldsymbol{\Omega}_{n\mathbf{p}} - e(\mathbf{v}_{\mathbf{p}}\cdot\boldsymbol{\Omega}_{\mathbf{p}})\boldsymbol{B}\right)
\end{aligned}$$

$$\boldsymbol{j}_{CME} = \left[-e^2 \sum_n \int_{\boldsymbol{p}} (\boldsymbol{v}_{n\boldsymbol{p}} \cdot \boldsymbol{\Omega}_{n\boldsymbol{p}}) f_{n\boldsymbol{p}}\right] \boldsymbol{B}$$

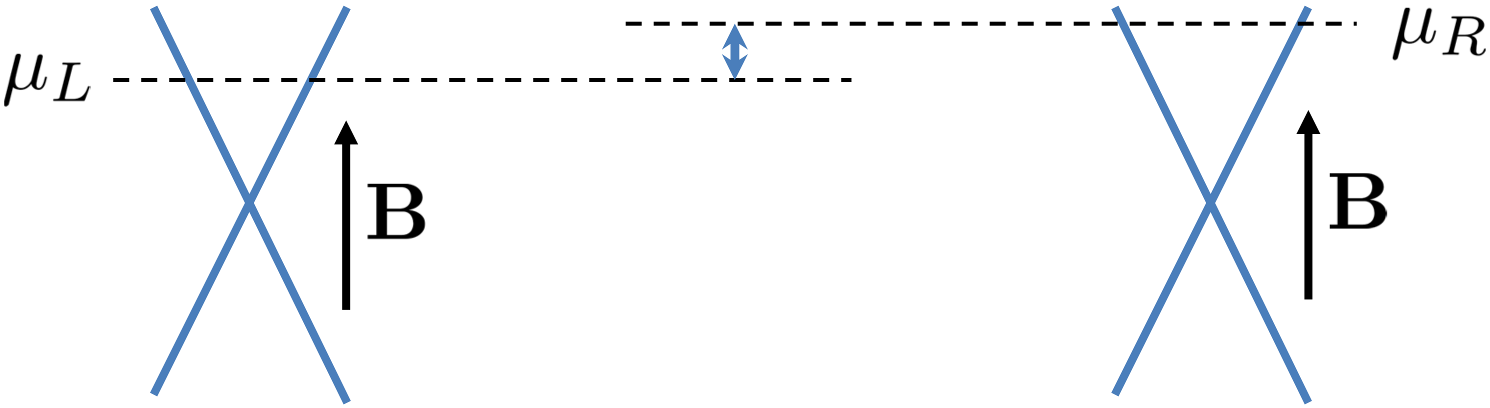
Looks like a "Fermi sea" current

However, using $\boldsymbol{v}_{n\boldsymbol{p}} = \partial_{\boldsymbol{p}} \epsilon_{n\boldsymbol{p}}$ and integrating by parts one arrives at

$$\begin{aligned}
\boldsymbol{j}_{CME} &= \left[e^2 \sum_n \int_{\boldsymbol{p}} (\epsilon_{n\boldsymbol{p}} \boldsymbol{\nabla}_{\boldsymbol{p}} \cdot \boldsymbol{\Omega}_{n\boldsymbol{p}}) f_{n\boldsymbol{p}} + \epsilon_{n\boldsymbol{p}} \boldsymbol{\Omega}_{n\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f_{n\boldsymbol{p}}\right] \boldsymbol{B} \\
&= \left[\frac{e^2}{4\pi^2} \sum_W \mu_W Q_W\right] \boldsymbol{B}
\end{aligned}$$

Berry monopoles are required for *static* CME

CME in a Weyl semimetal



$$\mathbf{j}_L^{CME} = \frac{e^2\mu_L}{4\pi^2}\mathbf{B}$$

(Vilenkin, 1980)

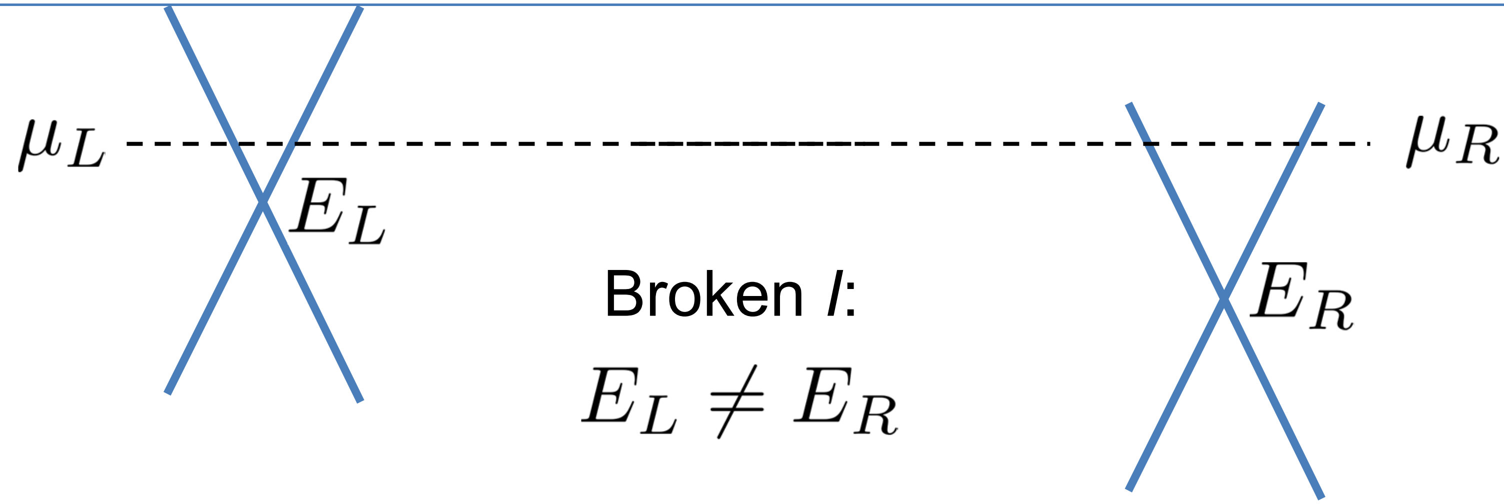
$$\mathbf{j}_R^{CME} = -\frac{e^2\mu_R}{4\pi^2}\mathbf{B}$$

$$\mathbf{j}_{\omega=0}^{CME} = \frac{e^2(\mu_L-\mu_R)}{4\pi^2}\mathbf{B}$$

$$\mathbf{j}_{\omega\neq 0}^{CME} = \frac{e^2(\mu_L-\mu_R)}{12\pi^2}\mathbf{B}$$

(Kharzeev, Warringa, 2009;
Son, Yamamoto, 2013)

There is only dynamic CME in equilibrium crystals



$$\mathbf{j}^{CME}_{\omega=0} = 0$$

(Zhou, Jiang, Niu, Shi, Chin. Phys. Lett., 2013; Vazifeh, Franz, PRL, 2013)

Physical reason: $\boldsymbol{j} \propto \boldsymbol{B}$ implies $\boldsymbol{M} \propto \boldsymbol{A}$ in equilibrium

(Levitov, Nazarov, Eliashberg, JETP 1985)

For $\mathbf{j}^{CME}_{\omega\neq 0}$ see Chen, Wu, Burkov, PRB, 2013
Chang, Yang PRB 2015;
Ma, Pesin, PRB 2015;
Zhong, Moore, Souza, PRL 2016

The chiral anomaly

$$\partial_t f_{\boldsymbol{p}} + \dot{\boldsymbol{p}} \partial_{\boldsymbol{p}} f_{eq} = \hat{I}_{st}^{intra}$$

$$\begin{aligned} \dot{\mathbf{p}} &= \frac{1}{D_{\boldsymbol{B}}} \left(e\boldsymbol{E} + e\mathbf{v}_{\mathbf{p}} \times \mathbf{B} - e^2 (\boldsymbol{E} \cdot \boldsymbol{B}) \boldsymbol{\Omega}_{\mathbf{p}} \right) \\ \dot{\mathbf{r}} &= \frac{1}{D_{\boldsymbol{B}}} \left(\mathbf{v}_{\mathbf{p}} - e\boldsymbol{E} \times \boldsymbol{\Omega}_{n\mathbf{p}} - e(\mathbf{v}_{\mathbf{p}} \cdot \boldsymbol{\Omega}_{\mathbf{p}}) \boldsymbol{B} \right) \end{aligned}$$

Equation for the density in a given valley: $\rho_W = e \int_{\boldsymbol{p}} D_{\boldsymbol{B}} f_{\boldsymbol{p}}$

$$\partial_t \rho_W = e^3 (\boldsymbol{E} \cdot \boldsymbol{B}) \int_{\boldsymbol{p}} \boldsymbol{\Omega}_{\boldsymbol{p}} \partial_{\boldsymbol{p}} f_{eq} = e^3 (\boldsymbol{E} \cdot \boldsymbol{B}) \int_{\boldsymbol{p}} (\boldsymbol{\Omega}_{\boldsymbol{p}} \cdot \boldsymbol{v}_{\boldsymbol{p}}) \partial_{\varepsilon_{\boldsymbol{p}}} f_{eq}$$

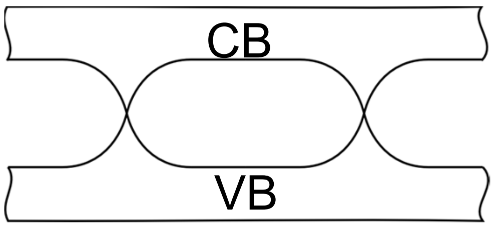
$$= \frac{e^3}{4\pi^2} Q_W \boldsymbol{E} \cdot \boldsymbol{B}$$

Total charge near an individual Weyl point is not conserved

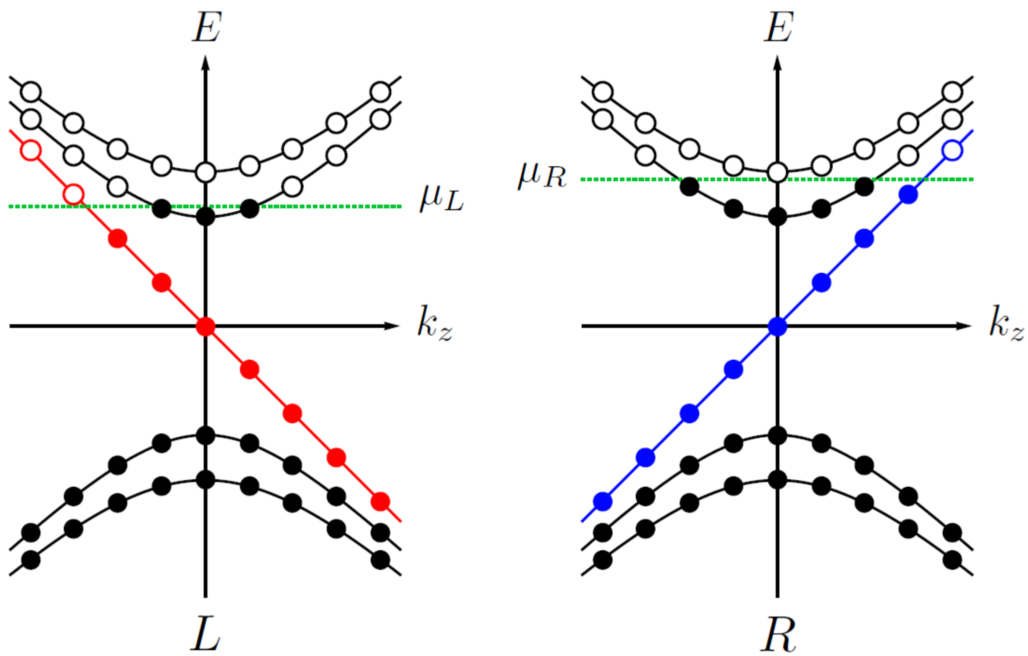
The net charge conservation is ensured by “Berry neutrality”:

$$\sum_W Q_W = 0$$

LL interpretation: chiral anomaly



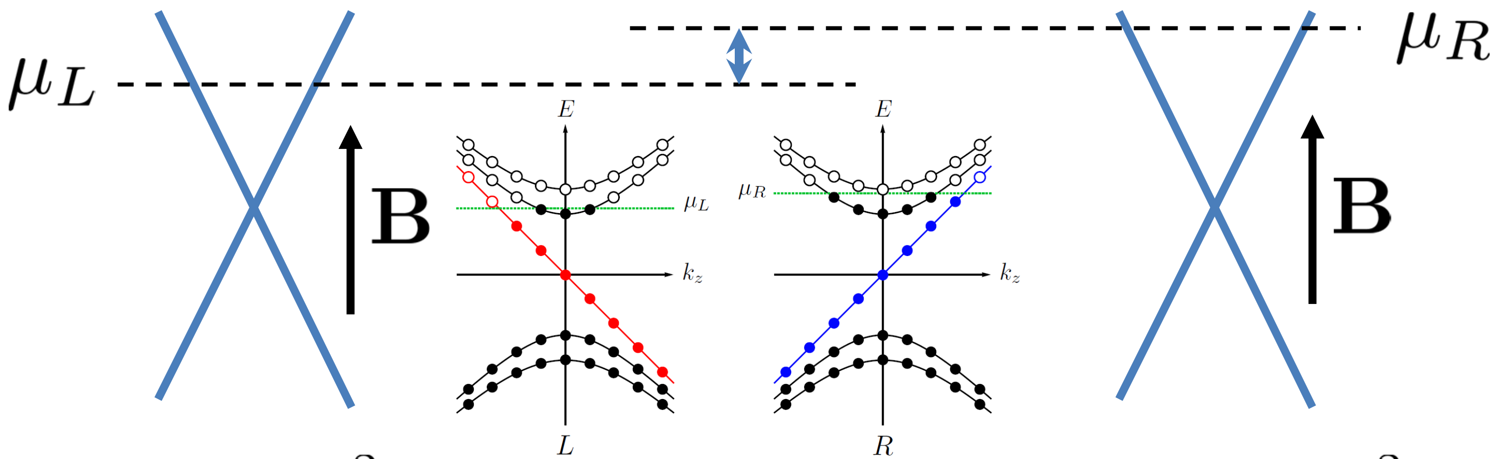
$$\Rightarrow \quad \vec{B} = (0,0,B),$$
$$H = \pm v \left[\underbrace{\vec{\sigma}_\perp (\vec{p}_\perp - e\vec{A})}_{\text{“graphene”}} + \underbrace{\sigma_z p_z}_{\text{“gap”}} \right]$$



$$\boxed{\dot{N}_R - \dot{N}_L = \frac{e^2}{2\pi^2\hbar^2 c}\mathbf{E}\cdot\mathbf{B}}$$ **“3D chiral anomaly”**

(S. L. Adler, 1969 ; J. S. Bell and R. Jackiw, 1969; Nielsen&Ninomiya, 1983)

LL interpretation: CME



$$\mathbf{j}_L^{CME} = \frac{e^2 \mu_L}{4\pi^2}\mathbf{B}$$

(Vilenkin, 1980)

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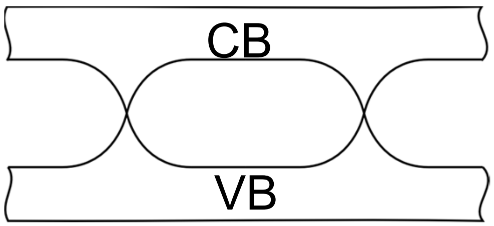
$$\mathbf{j}_{\omega=0}^{CME} = \frac{e^2(\mu_L - \mu_R)}{4\pi^2}\mathbf{B}$$

$$\mathbf{j}_{\omega\neq 0}^{CME} = \frac{e^2(\mu_L - \mu_R)}{12\pi^2}\mathbf{B}$$

(Kharzeev, Warringa, 2009;
Son, Yamamoto, 2013)

Recap: chiral anomaly



$\Rightarrow$

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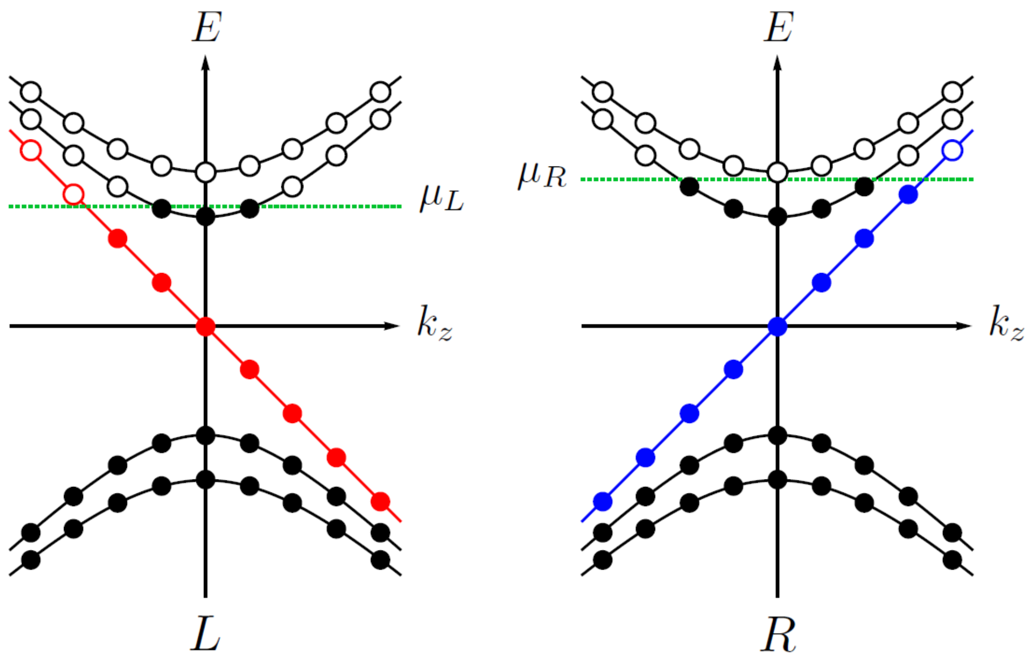
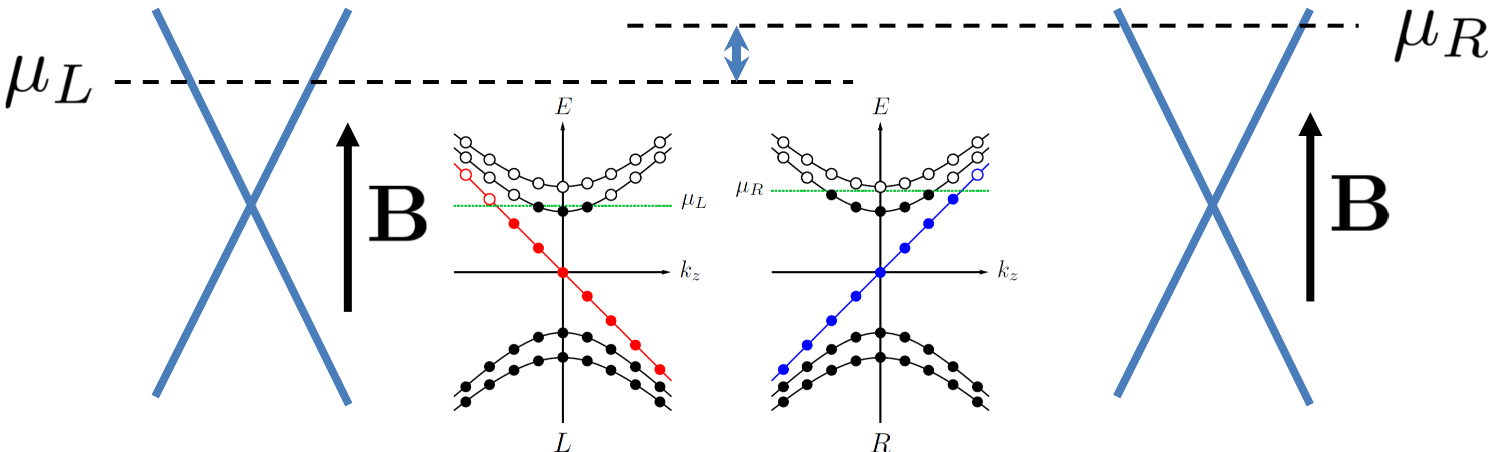


$$\boxed{\dot{N}_R - \dot{N}_L = \frac{e^2}{2\pi^2\hbar^2 c}\mathbf{E}\cdot\mathbf{B}}$$ **“3D chiral anomaly”**

(S. L. Adler, 1969 ; J. S. Bell and R. Jackiw, 1969; Nielsen&Ninomiya, 1983)

Recap: CME



$$\mathbf{j}_L^{CME} = \frac{e^2\mu_L}{4\pi^2}\mathbf{B}$$

(Vilenkin, 1980)

$$\mathbf{j}_R^{CME} = -\frac{e^2\mu_R}{4\pi^2}\mathbf{B}$$

$$\mathbf{j}_{\omega=0}^{CME} = \frac{e^2(\mu_L - \mu_R)}{4\pi^2}\mathbf{B}$$

$$\mathbf{j}_{\omega\neq0}^{CME} = \frac{e^2(\mu_L - \mu_R)}{12\pi^2}\mathbf{B}$$

(Kharzeev, Warringa, 2009;
Son, Yamamoto, 2013)

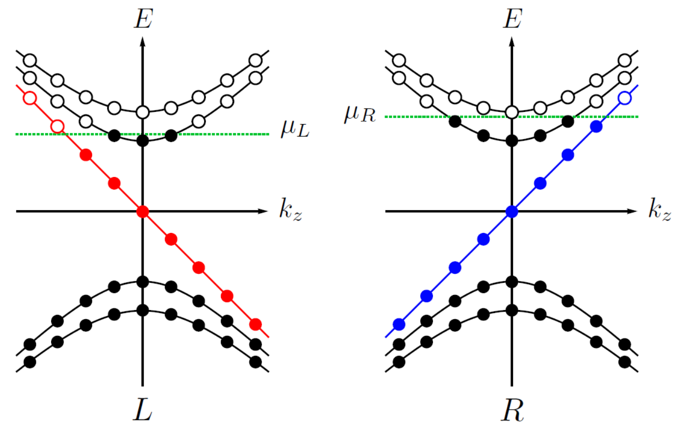
“Anomalous” transport theory in WS

(for a hydrodynamic description see Lucas, Richardson, Sachdev, PNAS 2016)

The currents include the chiral modes contributions:

$$\mathbf{j}^{R,L} = -\frac{\sigma}{e}\boldsymbol{\nabla}\mu_{\text{ec}}^{R,L} \pm \frac{e^2\mathbf{B}}{4\pi^2\hbar^2 c}\mu^{R,L}.$$

$$\mu_{\text{ec}}^{R,L} = \mu^{R,L} + e\phi$$



The continuity equations include the anomalous divergences:

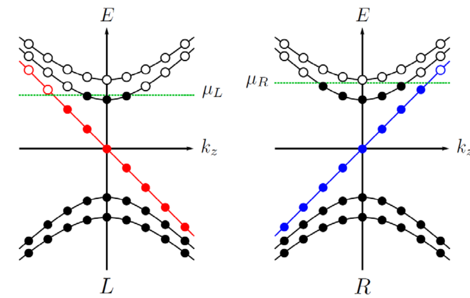
$$\boldsymbol{\nabla}\cdot\mathbf{j}^{R,L} + \partial_t\rho^{R,L} = \pm\frac{e^3}{4\pi^2\hbar^2 c}\mathbf{E}\cdot\mathbf{B}$$

The final stationary transport equations contain only $\mu_{\text{ec}}^{R,L}$

$$\boxed{-\frac{\sigma}{e}\nabla^2\mu_{\text{ec}}^{R,L} \pm \frac{e^2}{h^2}\boldsymbol{B}\cdot\boldsymbol{\nabla}\mu_{\text{ec}}^{R,L} = \mp\frac{e\nu_{3\text{D}}}{2\tau_v}(\mu_{\text{ec}}^{R} - \mu_{\text{ec}}^{L})}$$

Negative magnetoresistance from the chiral anomaly

(Son, Spivak, PRB 2012)



For clarity: $-\boldsymbol{\nabla}\mu_{ec} \rightarrow e\boldsymbol{E}$

Use chiral anomaly to generate imbalance:

$$\frac{e^3}{4\pi^2}B_z E_z = \frac{e\nu_{\mathrm{3D}}}{2\tau_v}(\mu^R - \mu^L) \Rightarrow \mu^R - \mu^L = \frac{e^2}{2\pi^2}\frac{\tau_v}{\nu_{3D}}B_z E_z$$

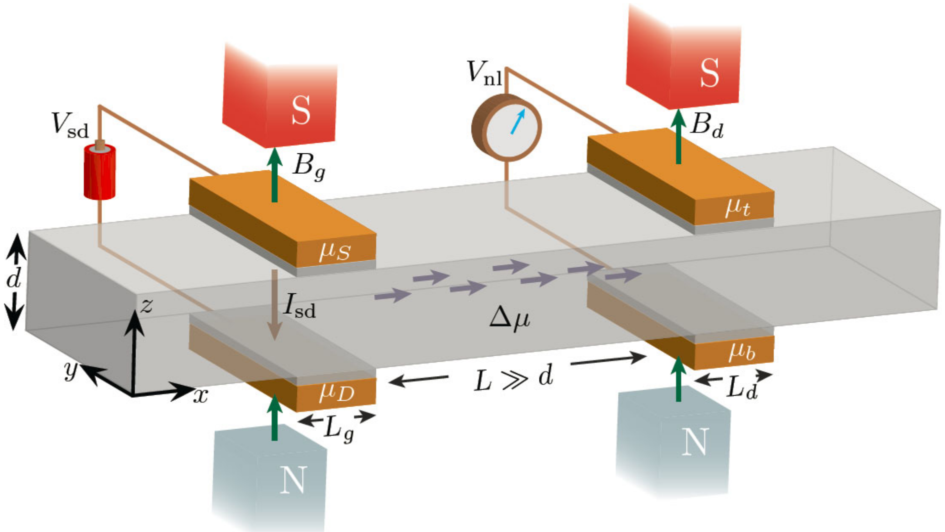
Convert the imbalance into “more conductivity” by the CME:

$$\delta j = \frac{e^2}{4\pi^2}(\mu^R - \mu^L) \Rightarrow \delta\sigma_{zz} = \frac{e^4}{8\pi^4}\frac{\tau_v}{\nu_{3D}}B_z^2$$

$$\frac{\delta\sigma_{zz}}{|\sigma_z z(B) - \sigma_{zz}(0)|} \sim \frac{\tau_v}{\tau}\frac{1}{\mu^2\tau^2}$$ can be large

For a discussion of experimental issues, see Liang et al., PRX 2018

Non-local transport from chiral anomaly/CME



$$\frac{|V_{\rm nl}(x)|}{V_{\rm SD}} \propto e^{-x/\ell_v}, \quad \ell_v = \sqrt{D\tau_v} \gg d$$

S. Parameswaran, T. Grover, D. Abanin, **DP,** A. Vishwanath, PRX 4, 031035 (2014)

Measurement:
C. Zhang, et al
Nature, 2017