Quantum mechanics of fluids in solids (II)

Dima Pesin
University of Virginia, Charlottesville, VA, USA

El Hydro School, SRitp/WIS
01/10/2018

So far mostly phenomena related to the Berry curvature

Is there anything else of interest?

Conceptual question

Pauli: spin is not self-rotation!

The physical interpretation of Pauli's "degree of freedom" was initially unknown. Ralph Kronig, one of Landé's assistants, suggested in early 1925 that it was produced by the self-rotation of the electron. When Pauli heard about the idea, he criticized it severely, noting that the electron's hypothetical surface would have to be moving faster [illegible]d of light in order for it to rotate quickly enough to produce the necessary angular momentum. This would violate the theor[illegible]ativity. [illegible] due to Pauli's criticism, Kronig decided not to publish his idea.

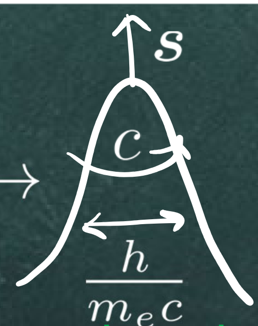
LL vol 3, "The current densit[illegible] a magnetic field":

Comparing this expression with ([illegible]5.1), we find the following expression for the current density:

$$\mathbf{j} = \frac{ie\hbar}{2m}[(\nabla\Psi^*)\Psi - \Psi^*\nabla\Psi] - \frac{e^2}{mc}\mathbf{A}\Psi^*\Psi + (\mu/s)c\,\mathbf{curl}(\Psi^*\hat{\mathbf{s}}\Psi). \quad (115.4)$$

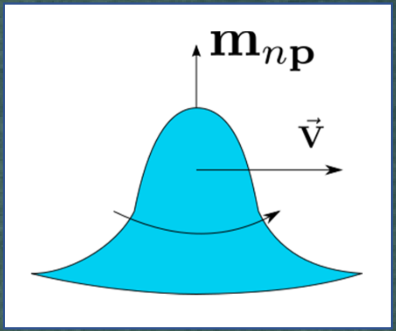
Resolution:

$$\boldsymbol{m} = g\frac{e\hbar}{2m_e}\boldsymbol{s} \leftrightarrow$$



See also: effective g-factor in semiconductors

Intrinsic orbital moment



Wave packet in a single band cannot have zero width, hence can be assigned self-rotation.

Naturally, there is a magnetic moment associated with that self-rotation (a mechanical moment can be there, too)

$$\mathbf{m}_{n\mathbf{p}} = \frac{ie}{2}\langle\partial_{\mathbf{p}} u_{n\mathbf{p}}| \times (h_{\mathbf{p}} - \epsilon_{n\mathbf{p}})|\partial_{\mathbf{p}} u_{n\mathbf{p}}\rangle,$$

Orbital moments for Weyl fermions

Intrinsic magnetic moment (conduction band):

$$\mathbf{m}_{\boldsymbol{p}} = \frac{ie}{2}\langle \partial_{\boldsymbol{p}} u_{\boldsymbol{p}} | \times (h_{\boldsymbol{p}} - \epsilon_{\boldsymbol{p}}) | \partial_{\boldsymbol{p}} u_{\boldsymbol{p}} \rangle = \frac{ev\hbar}{2p} \boldsymbol{e}_{\boldsymbol{p}}$$

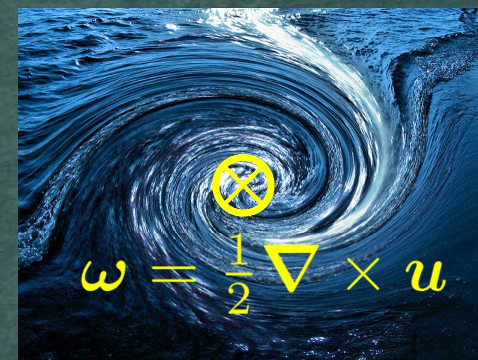
Intrinsic mechanical moment:

$$\boldsymbol{L}_p = \langle (\boldsymbol{r} - \boldsymbol{r}_c) \times (\boldsymbol{p} - \boldsymbol{p}_c) \rangle + \frac{\hbar}{2}\langle \boldsymbol{\sigma} \rangle = \frac{\hbar}{2} \boldsymbol{e}_{\boldsymbol{p}}$$

Gyromagnetic ratio:

$$\frac{m_p}{L_p} = \frac{ev}{p} = \frac{ev^2}{\varepsilon_p} = \frac{e}{m_e}\Big|_{m_e \to \varepsilon_p / v^2}$$

Chiral vortical effect of Weyl fermions



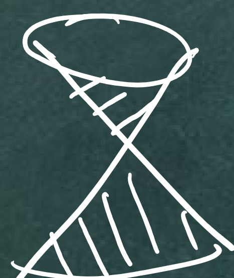
(broken I)

$$\boldsymbol{j} = en\boldsymbol{u} + \tfrac{1}{2}\lambda_{\mathrm{cve}}\boldsymbol{\nabla}\times\boldsymbol{u} \equiv en\boldsymbol{u} + \lambda_{\mathrm{cve}}\boldsymbol{\omega}$$

$$\lambda_{\mathrm{cve}} = ?$$

Model of choice: single species of Weyl fermions

$$H = v\boldsymbol{\sigma}\cdot\boldsymbol{p},$$



-- "Conduction band" only

The CVE current does not appear in more conventional models, e.g. chiral suspension in a non-chiral fluid (Andreev, Son, Spivak 2009)

CVE: rotating frame (Stephanov, Yin, PRL 2012)

Switch to a reference frame rotating with the fluid: Coriolis force

$F_{\rm cor} = 2m\dot{\boldsymbol{r}} \times \boldsymbol{\omega} \to \frac{2\varepsilon}{v^2}\dot{\boldsymbol{r}} \times \boldsymbol{\omega}$, or use $\hat{H} = v\boldsymbol{\sigma}\cdot\boldsymbol{p} - \boldsymbol{\omega}\cdot(\boldsymbol{r}\times\boldsymbol{p} + \frac{1}{2}\boldsymbol{\sigma})$

This looks like an “energy-dependent” B-field:

$$e\boldsymbol{B} = \frac{2\varepsilon}{v^2}\boldsymbol{\omega}$$

Use the CME formula $j_{CME} = \left[-e^2 \sum_n \int_{\boldsymbol{p}} (\boldsymbol{v}_{n\boldsymbol{p}}\cdot\boldsymbol{\Omega}_{n\boldsymbol{p}}) f_{n\boldsymbol{p}}\right] \boldsymbol{B}$ to obtain

$$\boxed{\boldsymbol{j}_{\rm cve} = \boldsymbol{\omega}\left[-e\int_{\boldsymbol{p}} (\boldsymbol{\Omega}\boldsymbol{v})\frac{2\varepsilon_p}{v^2} f_{\boldsymbol{p}}\right] = e\frac{p_F^2}{4\pi^2}\boldsymbol{\omega}}$$

CVE: lab frame

- We need the local-equilibrium distribution function up to the linear order in gradients of the macroscopic velocity.
- For the model at hand – isotropic Weyl cone – we can find the d.f. based on the angular momentum conservation.

For uniform macroscopic translation and rotation,

$$\boldsymbol{u} = \boldsymbol{u}_0 + \boldsymbol{\omega} \times \boldsymbol{r}, \ \boldsymbol{\omega} = \frac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{u}$$

the d.f. is given by

$$f_p = f_{eq}(\beta[\varepsilon_p - (\boldsymbol{u}_0 + \boldsymbol{\omega} \times \boldsymbol{r}) \cdot \boldsymbol{p} - \boldsymbol{\omega} \cdot \boldsymbol{s}_p]) \rightarrow \text{(local equilibrium)}$$

$$f_{eq}(\beta[\varepsilon_p - \boldsymbol{u}(\boldsymbol{r}) \cdot \boldsymbol{p} - \tfrac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{u}(\boldsymbol{r}) \cdot \boldsymbol{s}_p])$$

CVE in the lab frame: two currents

$$\boldsymbol{j}_{\rm cve} = \boldsymbol{j}_{\rm bal} + \boldsymbol{j}_{\rm mag},$$ (ballistic+magnetization)

$$\boldsymbol{j}_{\rm bal} = e\int_{\boldsymbol{p}} \boldsymbol{v}_{\boldsymbol{p}} f_{l.e.}(\boldsymbol{r}) = \int_p e\vec{v}_p \cdot (-\vec{\omega}\cdot\vec{\Sigma}_p)\,\partial_{\varepsilon_p} f_{eq}$$

$$= \frac{e}{8\pi^3}\cdot\frac{4\pi}{3}\cdot\int_0^{p_F} dp\cdot p^2\cdot v\cdot\frac{1}{2}\cdot\delta(vp-\mu)\,\vec{\omega} =$$

$$= \frac{1}{3}\, e\,\frac{p_F^2}{4\pi^2}\,\vec{\omega};$$

CVE in the lab frame: two currents

$$\boldsymbol{j}_{\rm cve} = \boldsymbol{j}_{\rm bal} + \boldsymbol{j}_{\rm mag},$$ (ballistic+magnetization)

$$\boldsymbol{j}_{\rm mag} = \boldsymbol{\nabla} \times \int_{\boldsymbol{p}} \boldsymbol{m}_{\boldsymbol{p}} f_{l.e.}(\boldsymbol{r}) = \vec{\nabla} \times \int_p \vec{m}_p \left(-\vec{u}\cdot\vec{p}\right) \partial_{\varepsilon_p} f_{eq} =$$

$$= \vec{\nabla} \times \frac{1}{8\pi^3} \cdot \frac{4\pi}{3} \cdot \int_0^{p_F} dp \cdot p^2 \cdot \frac{ev}{2p} \cdot p\, \delta(vp-\mu)\, \vec{u} =$$

$$= \frac{e}{3} \cdot \frac{p_F^2}{4\pi^2} \cdot \vec{\nabla} \times \vec{u} = \frac{2}{3} e \frac{p_F^2}{4\pi^2} \cdot \vec{\omega}$$

Weyl CVE results

$$\boldsymbol{j}^{lab}_{\rm cve} = \frac{1}{3}\boldsymbol{j}^{r.f.}_{\rm cve} + \frac{2}{3}\boldsymbol{j}^{r.f.}_{\rm cve} = \boldsymbol{j}^{r.f.}_{\rm cve} = e\frac{p_F^2}{4\pi^2}\boldsymbol{\omega}.$$

(does not get renormalized in ***Lorentz-inv*** interacting theory, Landsteiner, 2011)

does not hold

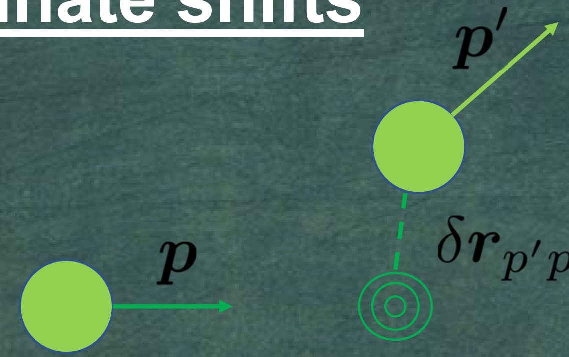
However:

- The CVE in the r.f. looks like a "Fermi sea" quantity, while the l.f. expression is a "Fermi surface" one. The two are connected by an integration by parts. Is this a general observation?
- How to get the right result in the lab frame without knowing StatMech?
- What is the generalization to the case where there is no angular momentum conservation?

All questions are essentially related to the "CVE in crystals" problem

Band geometry effects in collisions

Single-partile collisional coordinate shifts



"Side jump" **(Berger, 1970):**

Qualitative picture for a smooth impurity potential: displacement in the impurity's electric field due to the anomalous velocity

$$\delta \boldsymbol{r}_{\boldsymbol{p}\boldsymbol{p}'} = \int_{t_i}^{t_f} dt\, \boldsymbol{\Omega} \times \dot{\boldsymbol{p}} = \boldsymbol{\Omega} \times (\boldsymbol{p} - \boldsymbol{p}')$$

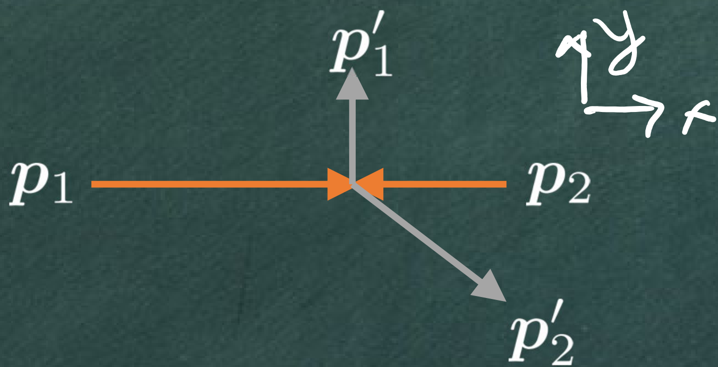
Full result, any weak impurity

$$\delta \boldsymbol{r}_{pp'} = \langle u_{\boldsymbol{p}} | i\partial_{\boldsymbol{p}} u_{\boldsymbol{p}} \rangle - \langle u_{\boldsymbol{p}'} | i\partial_{\boldsymbol{p}'} u_{\boldsymbol{p}'} \rangle - (\partial_{\boldsymbol{p}} + \partial_{\boldsymbol{p}'}) \mathrm{Arg} \langle u_{\boldsymbol{p}} | u_{\boldsymbol{p}'} \rangle$$

(Belinicher, Ivchenko, Sturman, 1982; Sinitsyn, MacDonald, Niu, 2007)

The need for two-particle shifts in two-particle collisions: the case of Weyl fermions

Naïve scattering diagram:

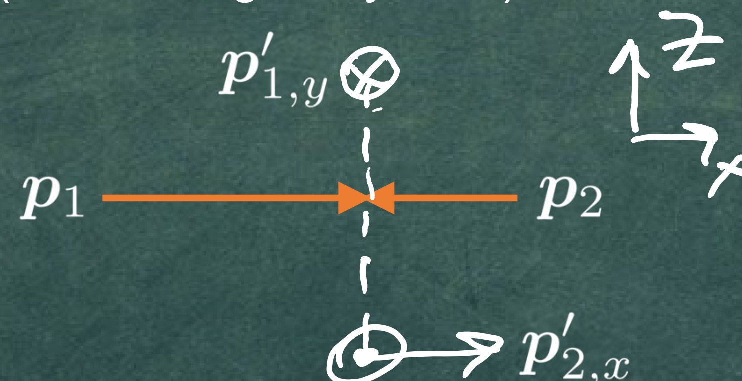


$$\boldsymbol{\ell}_{in} = 0,\ \ \boldsymbol{s}_{in} = 0$$

$$\boldsymbol{\ell}_{out} = 0,\ \ \boldsymbol{s}_{out} \neq 0$$

Something is wrong!

Reality (view along the y-axis)

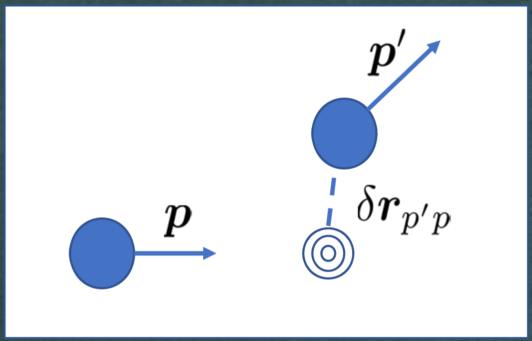


$$\boldsymbol{\ell}_{in} = 0,\ \ \boldsymbol{s}_{in} = 0$$

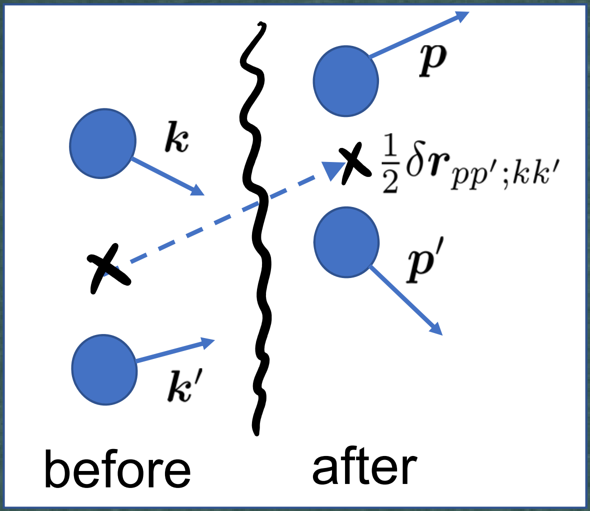
$$\boldsymbol{\ell}_{out} \neq 0,\ \ \boldsymbol{s}_{out} \neq 0,\ \ \boldsymbol{\ell}_{out} + \boldsymbol{s}_{out} = 0$$

Something is right!

Two-particle collisional coordinate shift



VS



- Individual shifts are ill-defined due to particle indistinguishability

- Symmetric combination – total shift – is well-defined:

$$R^{(-\infty)} = v_{\mathbf{k}} t + v_{\mathbf{k}'} t + \delta \mathbf{r}^{(-\infty)}$$

$$\longrightarrow \delta r_{\mathbf{pp}';\mathbf{kk}'} = \delta \mathbf{r}^{(+\infty)} - \delta \mathbf{r}^{(-\infty)}$$

$$R^{(+\infty)} = v_{\mathbf{p}} t + v_{\mathbf{p}'} t + \delta \mathbf{r}^{(+\infty)}$$

Main results for 2p coordinate shifts

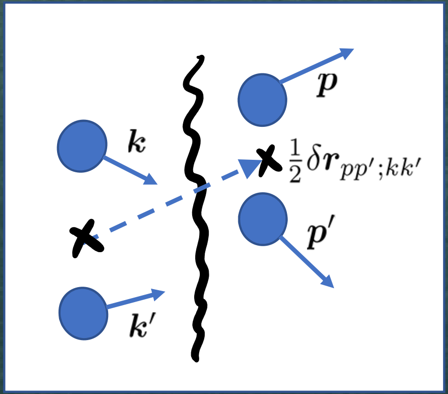
For $\langle \boldsymbol{p}\boldsymbol{p}'|\hat{V}_{e-e}|\boldsymbol{k}\boldsymbol{k}'\rangle \equiv V_{\boldsymbol{p}\boldsymbol{p}';\boldsymbol{k}\boldsymbol{k}'}$:

$$\begin{aligned}\delta \boldsymbol{r}_{\mathbf{pp'};\mathbf{kk'}} &= \langle u_{\boldsymbol{p}}|i\partial_{\boldsymbol{p}} u_{\boldsymbol{p}}\rangle + \langle u_{\boldsymbol{p}'}|i\partial_{\boldsymbol{p}'} u_{\boldsymbol{p}'}\rangle - \langle u_{\boldsymbol{k}}|i\partial_{\boldsymbol{k}} u_{\boldsymbol{p}}\rangle - \langle u_{\boldsymbol{k}'}|i\partial_{\boldsymbol{k}'} u_{\boldsymbol{k}'}\rangle \\ &- (\partial_{\boldsymbol{p}} + \partial_{\boldsymbol{p}'} + \partial_{\boldsymbol{k}} + \partial_{\boldsymbol{k}'})\arg V_{\boldsymbol{p}\boldsymbol{p}';\boldsymbol{k}\boldsymbol{k}'}\end{aligned}$$

Pesin, PRL 2018

Distinguishable particles: $\boldsymbol{p} \to \boldsymbol{k},\ \boldsymbol{p}' \to \boldsymbol{k}'$

$\delta \boldsymbol{r}_{\boldsymbol{p}\boldsymbol{p}';\boldsymbol{k}\boldsymbol{k}'} = \delta \boldsymbol{r}_{\boldsymbol{p};\boldsymbol{k}} + \delta \boldsymbol{r}_{\boldsymbol{p}';\boldsymbol{k}'}$ sum of individual 1p shifts



Weyl fermions with point interaction:

$$\delta \mathbf{r}_{\mathbf{kk'};\mathbf{pp'}} = -\frac{v}{2}\left(\frac{1}{\varepsilon_k} - \frac{1}{\varepsilon_{k'}}\right)\frac{\mathbf{e_k}\times\mathbf{e_{k'}}}{1-\mathbf{e_k}\cdot\mathbf{e_{k'}}} + \frac{v}{2}\left(\frac{1}{\varepsilon_p} - \frac{1}{\varepsilon_{p'}}\right)\frac{\mathbf{e_p}\times\mathbf{e_{p'}}}{1-\mathbf{e_p}\cdot\mathbf{e_{p'}}}$$

Individual shifts can be defined for point interaction, Lorentz-inv case, and zero angular momentum: Chen, Son, Stephanov, PRL 2015.

CVE in crystals and (much) more…

- Let us suppose one does not know statmech, but knows about the shifts. How would they arrive at the correct local-equilibrium d.f. to calculate the CVE?

- The key observation is that $f_{eq}(\beta[\varepsilon_p - \boldsymbol{u}(\boldsymbol{r})\cdot\boldsymbol{p}])$ does not nullify the collision integral anymore, the correction being $O(\partial\boldsymbol{u})$.

- In general, shifts transport the additive integrals of motion between liquid layers, hence contribute to the AHE (Pesin, PRL 2018), thermal Hall effect, viscosity: all seem to be the subject of the "***Anomalous hydrodynamics***".