What’s up with “Bad Metals”

A school is an invitation to pontificate

S. Kivelson and S. A. Kivelson, “Understanding Complexity,” Nature Phys. **14**, 426 (2018).

“Nominally, the purpose of a theory of a complex system is to supply an *understanding* of *essential* phenomena.”


Shoucheng Zhang
(1963 – 2018)

“Simple Metals” reflect perhaps the most “exotic” phases of matter

The Landau Fermi liquid theory of the metallic state ranks among the greatest achievements of the field.

* Extraordinary robustness despite large density of gapless modes:
existence of Fermi surface - $S = \gamma T + \ldots$

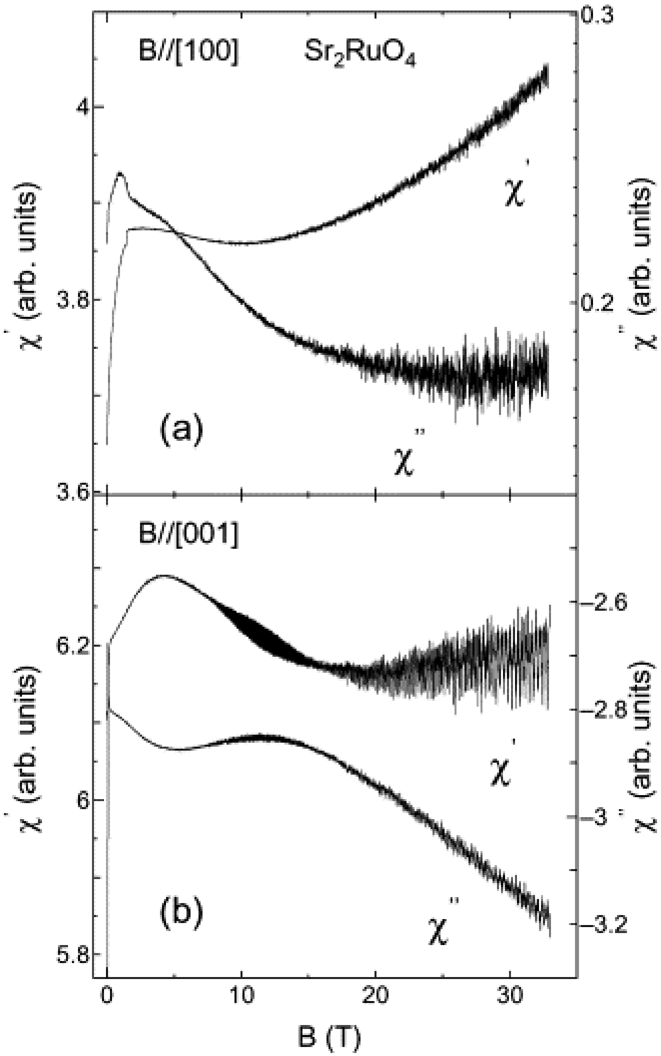
Marginal (BCS) instability to superconductivity

* Quantum dissipation – $\sigma(T) \rightarrow \sigma_0$ as $T \rightarrow 0$

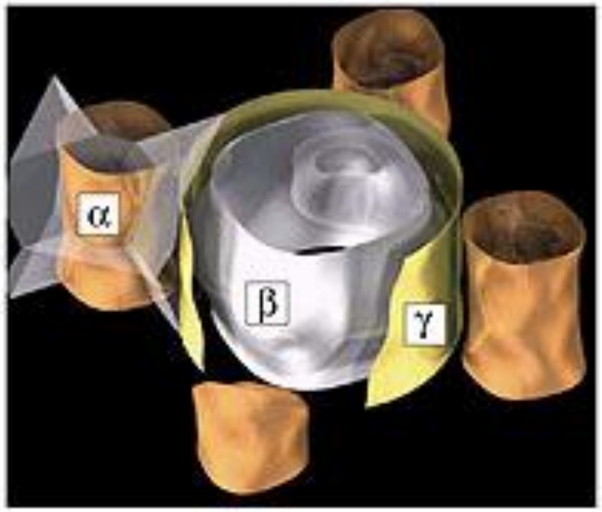
* Long-range entanglement: $S_{entangle} \sim L^{d-1} \log[k_F L]$

*Emergent quantum effects on all scales:
e.g. $M = B \; F(B/T, E_F/B)$

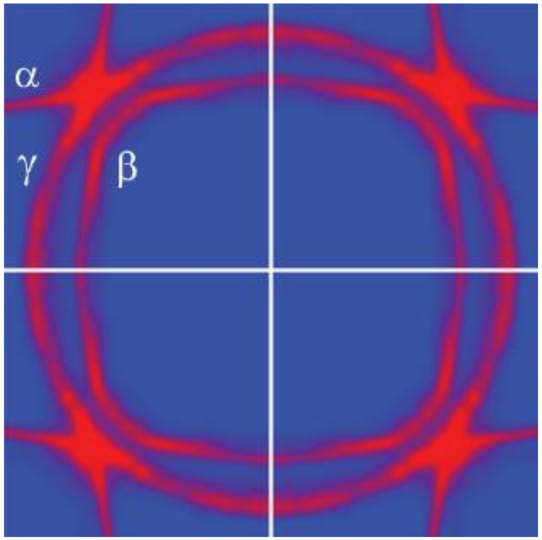
The Fermi Surface of Sr_2RuO_4 – It's real and measurable!



Quantum Oscillations



Inferred Fermi surface of Sr_2RuO_4



Angle resolved photoemision

Important resistivity scales from theory

$$\text{“Quantum of resistivity''} - \quad \rho_q \equiv \frac{h}{e^2}[a_B]^{d-2}$$

$$\rho_{2d} = 25.81 k\Omega \ ; \quad \rho_{3d} = 136.6 \mu\Omega\text{cm}$$

"Ab initio" theory of the resistivity of liquid and amorphous metals

$$\boxed{\rho = 136.6\mu\Omega cm}$$

$Be_{40}Ti_{50}Zr_{10}$ metallic glass : $\rho(4K) = 280\mu\Omega cm$ $\rho(600K) = 250\mu\Omega cm$

$\rho(\text{Hg at room temperature}) = 98\mu\Omega cm$

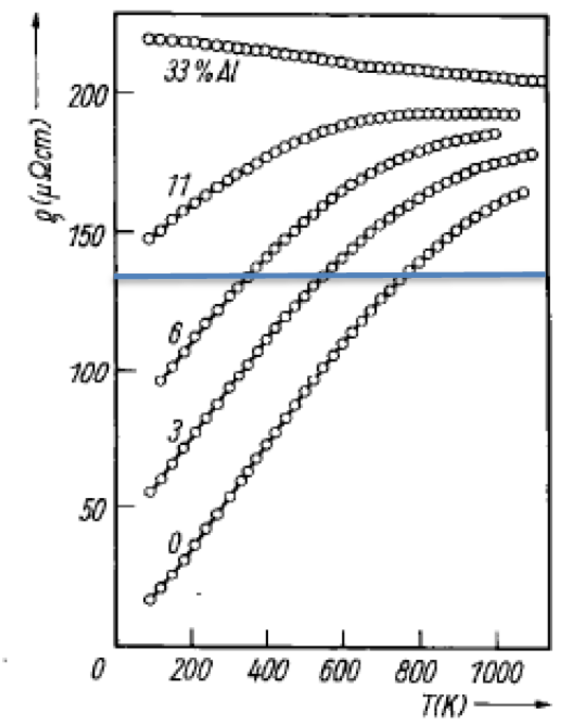


Fig. 3. Resistivity versus temperature for Ti and TiAl alloys containing 0, 3, 6, 11, and 33% Al

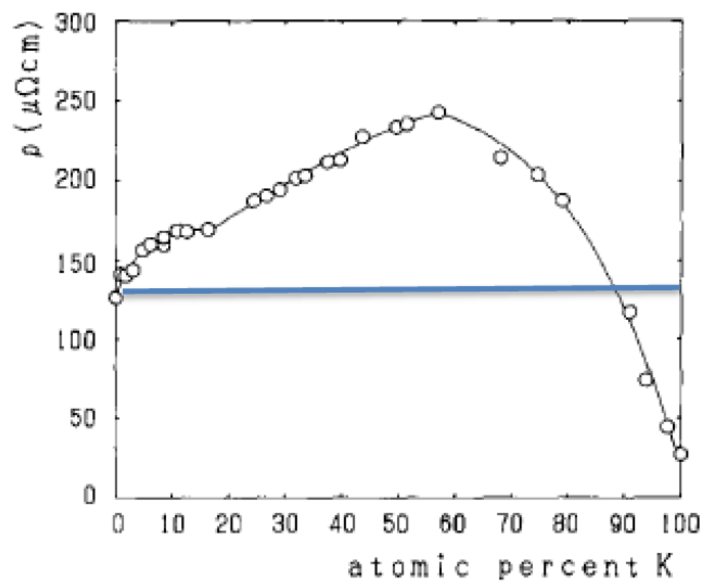


Fig 3. The concentration dependence of the electrical resistivity, ρ, of liquid K–Hg alloys at 573 K.

Important resistivity scales from theory (in 3d)

$$\text{“Quantum of resistivity''} - \quad \rho_q \equiv \frac{h}{e^2}[a_B]$$

$$\boxed{\text{Upper bounds on validity of theoretical approaches :}}$$

$$\rho_D = \frac{m}{e^2 n\tau} = \frac{3}{4}\left(\frac{h}{e^2}\right)\lambda_F \ (k_F\ell)^{-1}$$

$$\text{Landau – Fermi liquid theory : } \hbar/\tau \ll T \ \rightarrow \ \rho \ll \frac{3}{4}\left(\frac{h}{e^2}\right)\lambda_F \ \left(\frac{T}{E_F}\right)$$

$$\text{Boltzmann theory : } \hbar/\tau \ll E_F \ \rightarrow \ \rho \ll \frac{3}{4}\left(\frac{h}{e^2}\right)\lambda_F$$

$$\text{“Ioffe – Regel'' limit : } \ell \gg a \ \rightarrow \ \rho \ll \frac{3}{4}\left(\frac{h}{e^2}\right)\lambda_F\left(\frac{\Lambda}{E_F}\right)$$

Assuming some form of Drude theory …

$$\text{Landau – Fermi liquid theory}: \ \hbar/\tau \ll T \ \rightarrow \ \rho \ll \frac{3}{4}\left(\frac{h}{e^2}\right)\lambda_F \left(\frac{T}{E_F}\right)$$

$$\boxed{\hbar/\tau \ll T \quad \leftrightarrow \quad \ell \gg \hbar v_F/T \sim \lambda_F(E_F/T)}$$

$$\text{Boltzmann theory}: \ \hbar/\tau \ll E_F \ \rightarrow \ \rho \ll \frac{3}{4}\left(\frac{h}{e^2}\right)\lambda_F$$

$$\boxed{\hbar/\tau \ll E_F \quad \leftrightarrow \quad \ell \gg \lambda_F}$$

$$\text{“Ioffe – Regel''} \text{ limit}: \ \ell \gg a \ \rightarrow \ \rho \ll \frac{3}{4}\left(\frac{h}{e^2}\right)\lambda_F \left(\frac{\Lambda}{E_F}\right)$$

$$\boxed{\hbar/\tau \ll \Lambda \quad \leftrightarrow \quad \ell \gg a}$$

$$\text{“Incoherent} - \text{semi} - \text{quantum transport regime}''$$

$$E_F \gg T \gg \hbar\omega_0$$

$$\rho \sim \frac{h}{e^2}\lambda_F \quad \text{(or larger)}$$

$$\frac{d\rho}{dT} \gtrsim 0$$

$$\text{Why not MBL?}$$

$$\text{Resistivity saturation :} \quad \frac{1}{\rho(T)} \approx \frac{1}{\rho_0(T)} + \frac{1}{\rho_{sat}}$$

$$\rho_0(T) \approx \rho_q[\alpha_0 + \alpha_1 T]$$

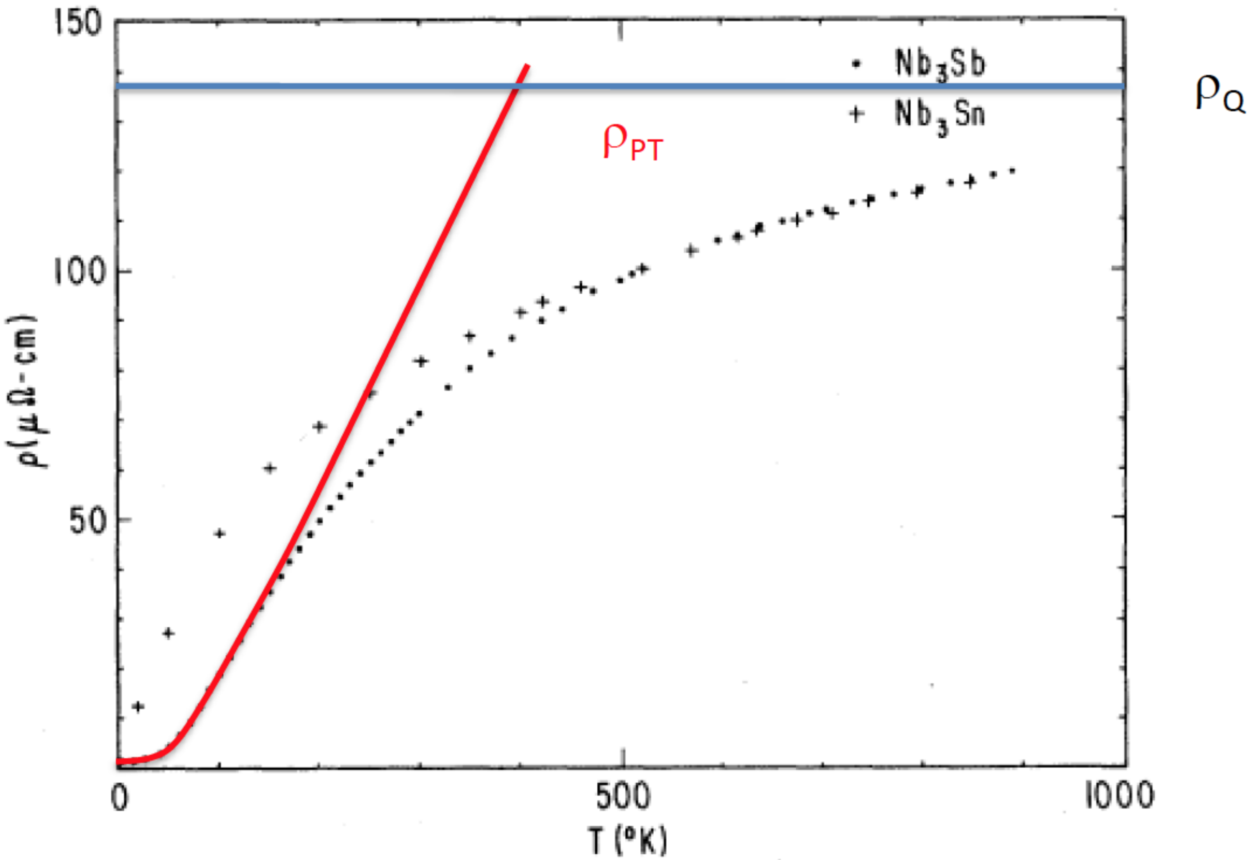
$$\rho_{sat} \sim \rho_q$$

$$\text{“Bad Metals}'' \quad \rho(T_{melt}) \gg \rho_q \;\&\; \frac{T}{\rho}\frac{d\rho}{dT} \sim 1$$

Gurvich, PRB (1981)

Emery and Kivelson, PRL (1995)

Resistivity saturation in good metals



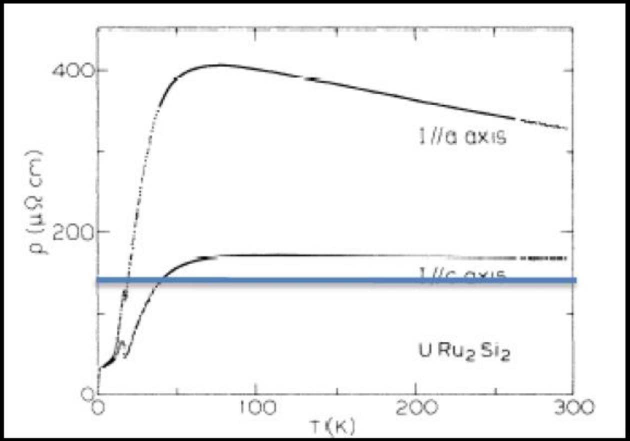
$$\rho(T) = \left[1/\rho_{PT}(T) + \sigma_{IR}\right]^{-1}$$

$$\sigma_{IR} \sim 1/\rho_Q$$

Heavy Fermions

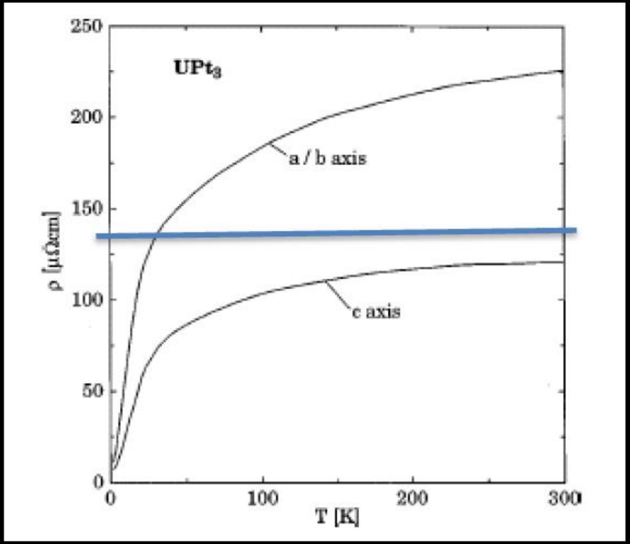
– the most extreme example of a saturating metal

URu_2Si_2



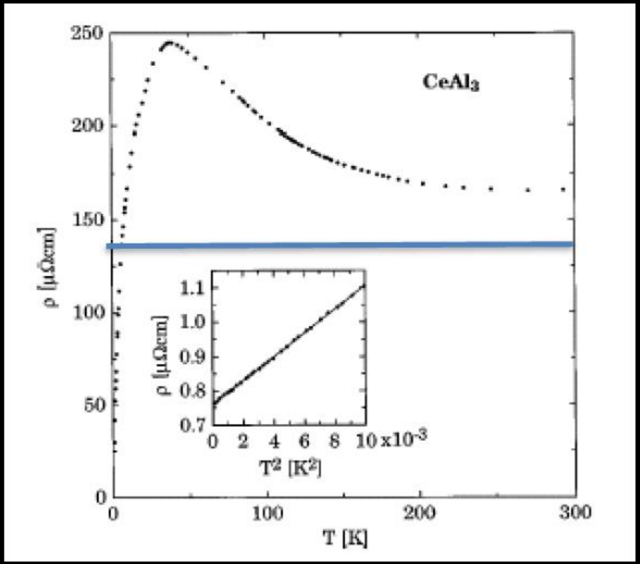
Palstra *et al.*, PRB (86)

UPt_3



de Visser *et al.*, JMMM (84)

Andres *et al.*, PRL (75)

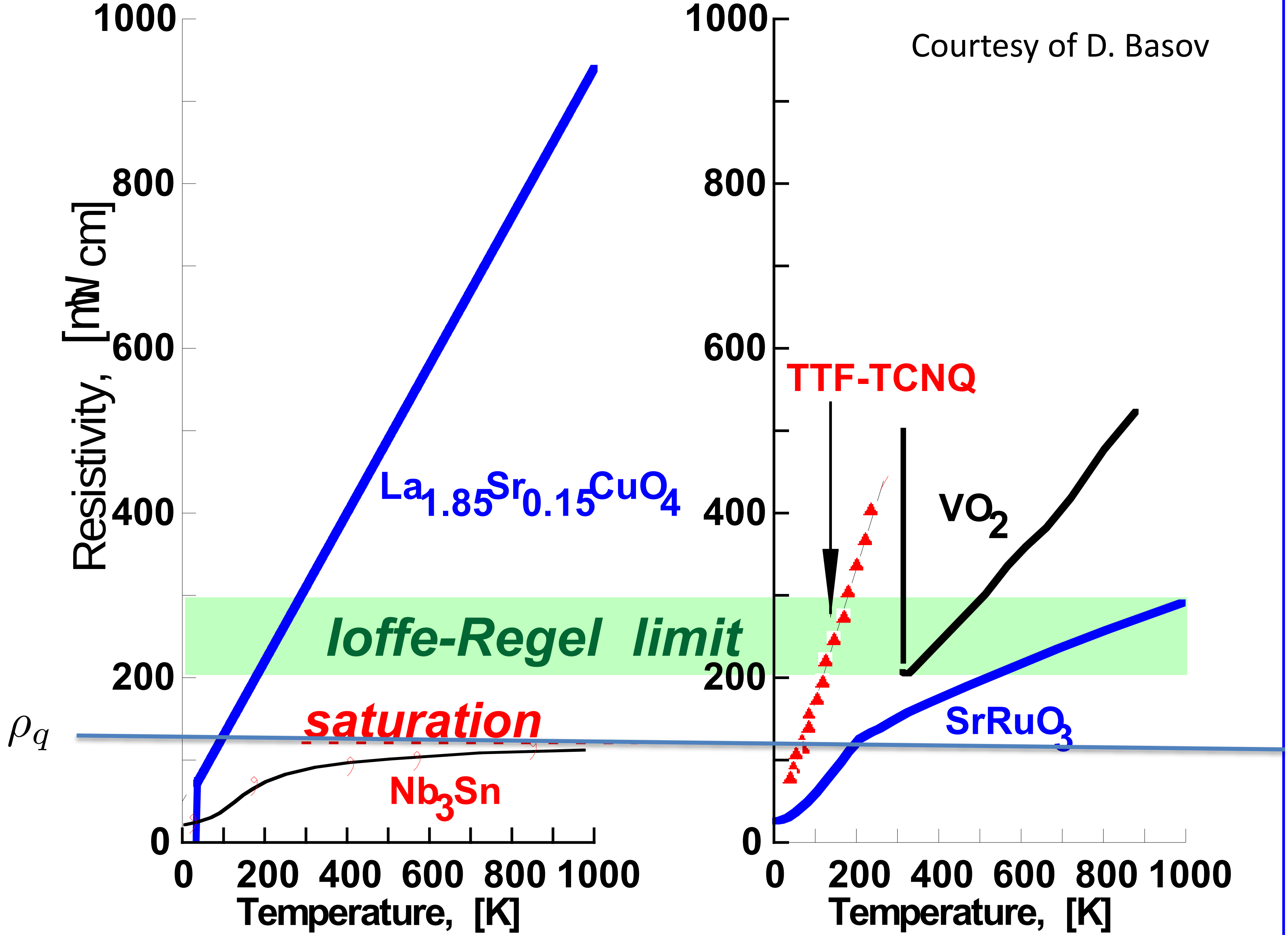


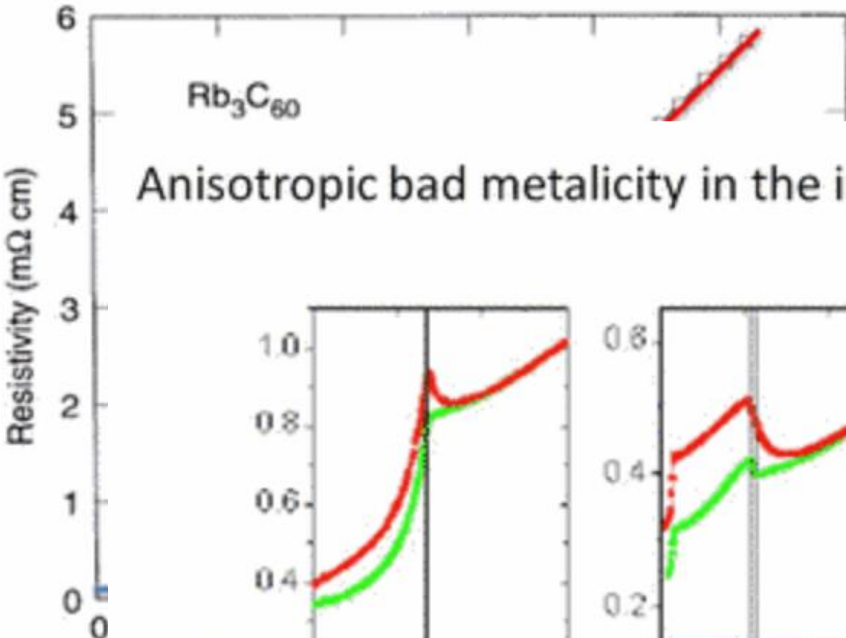
Slide courtesy of N. Hussey

“Bad Metals”

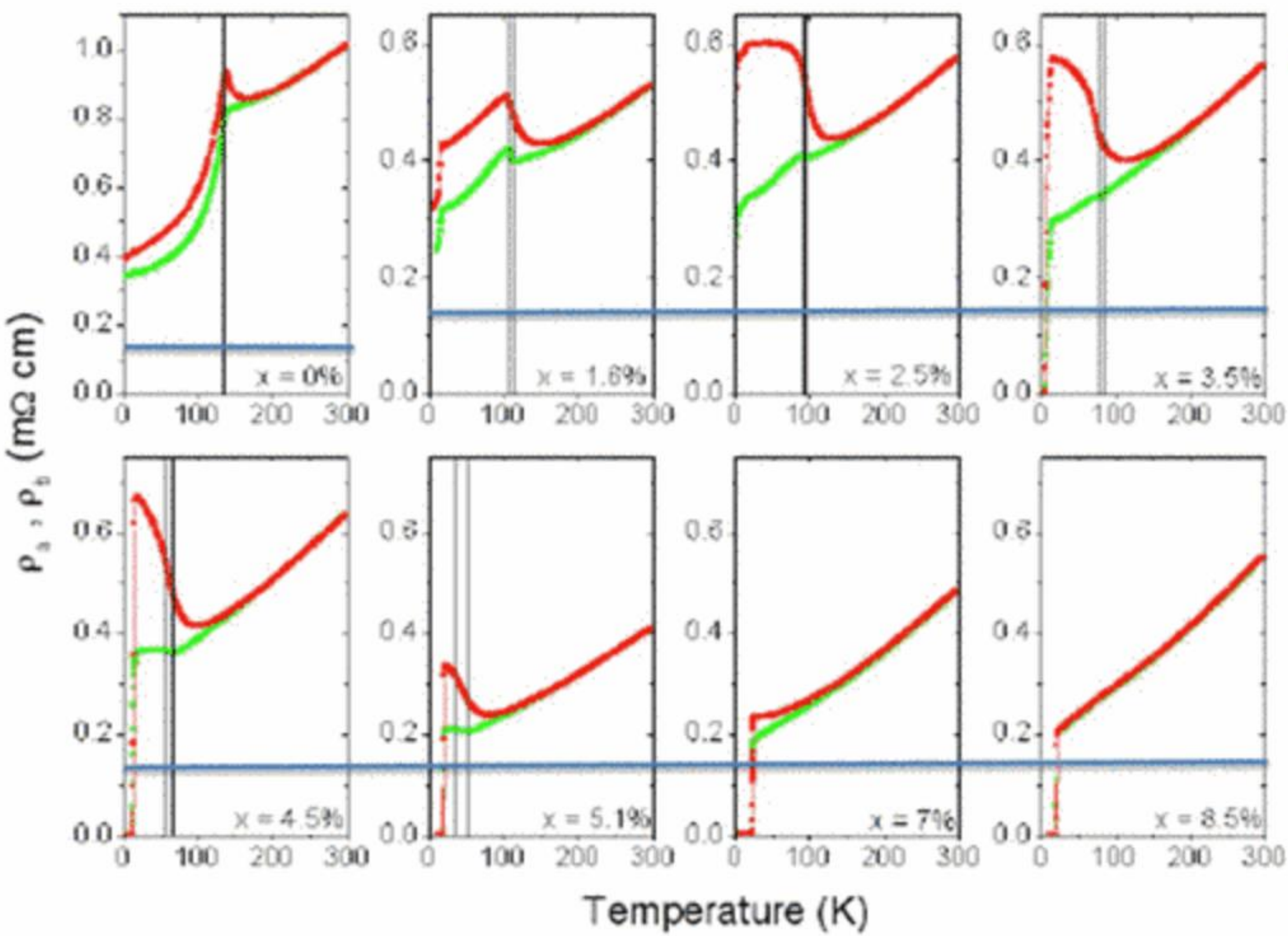
- Many correlated materials have “metallic” conductivities ($d\rho/dT > 0$) but at magnitudes that, at high T, rise well above ρ_Q.
  - Interpretted in terms of Boltzman transport, this would mean a mean-free path smaller than the Fermi wave-length – in violation of the “Ioffe-Regel” limit.
- Often $\rho \sim T$.
- Complete neglect of Mattheissen’s rule.

dc Transport in Synthetic Metals





Anisotropic bad metalicity in the iron-based high temperature superconductors



How desperate are we?

Quantum criticality as an organizing principle?

Strongly coupled incoherent fluid of the sort suggested by some AdS/CFT correspondence?

"At the least, however, holography can supply powerful metaphors, teaching physicists to think differently, leading to new questions to ask in experiments."

Approach to fundamental bounds on the rate of equilibration?

Novel Electron Hydrodynamic Regime?

B. Keimer, S.A. Kivelson, M.R. Norman, S. Uchida, J. Zaanen, Nature 518 (2015)

Might hydrodynamics come to the rescue?

Many “bad metals” are good crystals.

Bad metal regime appears relatively insensitive to sample quality – **maybe this remains true as disorder to 0**

Bad metals are “strongly correlated” **maybe that means l_{ee} = “small”**

Expressed in terms of viscosity, there is no obvious significance of ρ_q

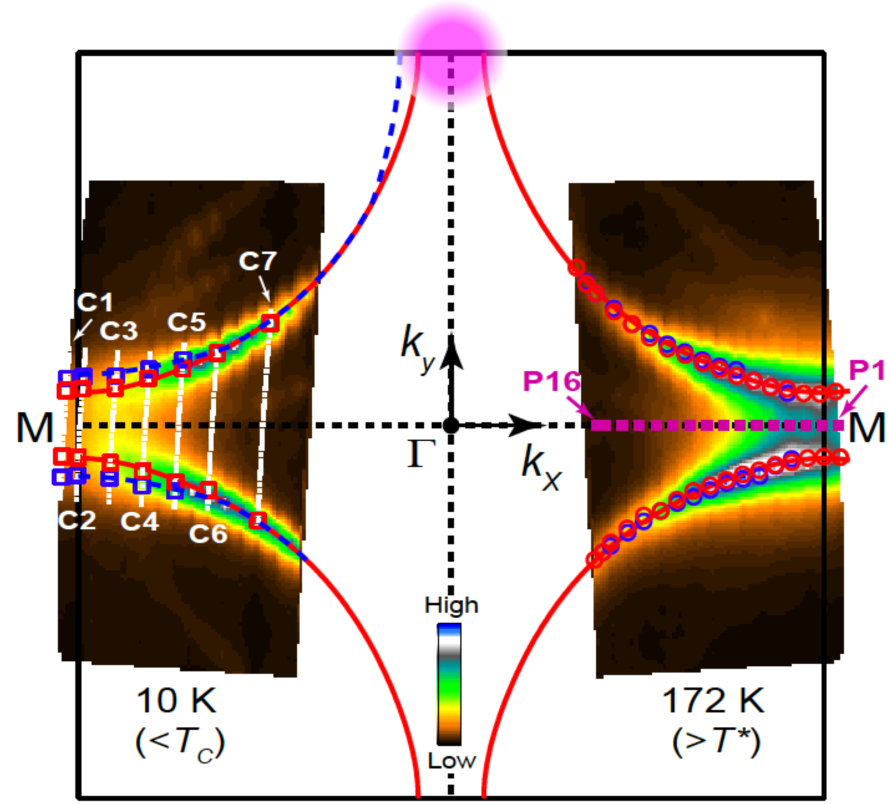
Might hydrodynamics come to the rescue?

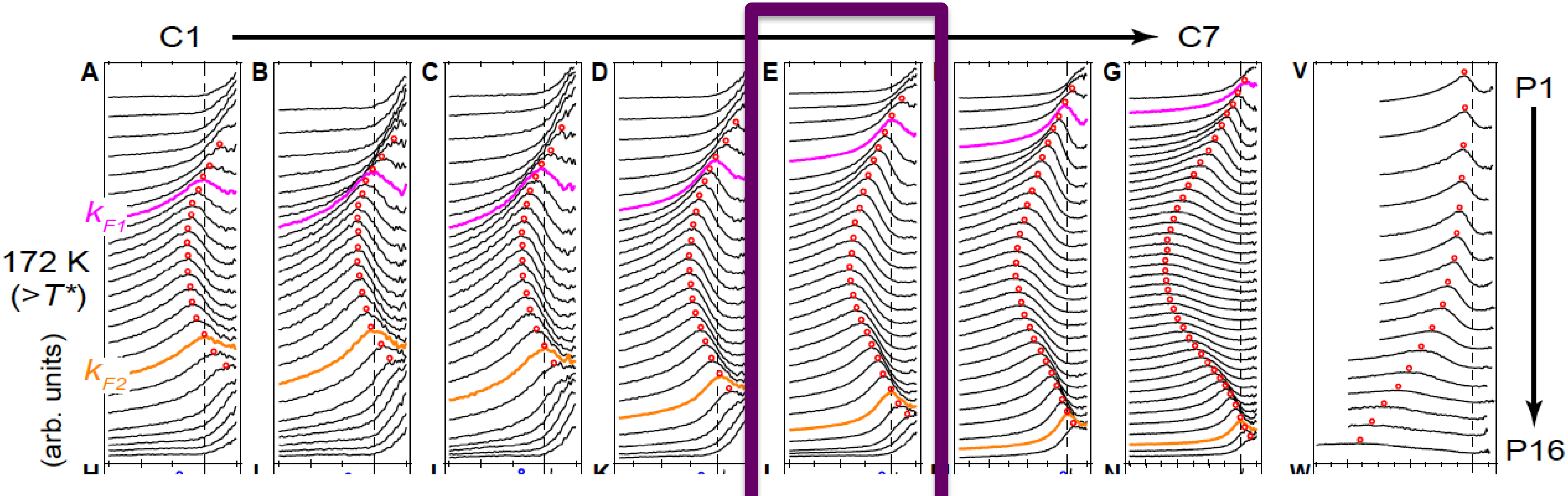
There are lots of phonons

There is both a priori and empirical evidence of strong electron-phonon coupling.

There is no reason to think that Umklapp is negligible, either for electrons or phonons.

"Normal" state (T > T*) ARPES from $Pb_{0.55}Bi_{1.5}Sr_{1.6}La_{0.4}CuO_{6+\delta}$



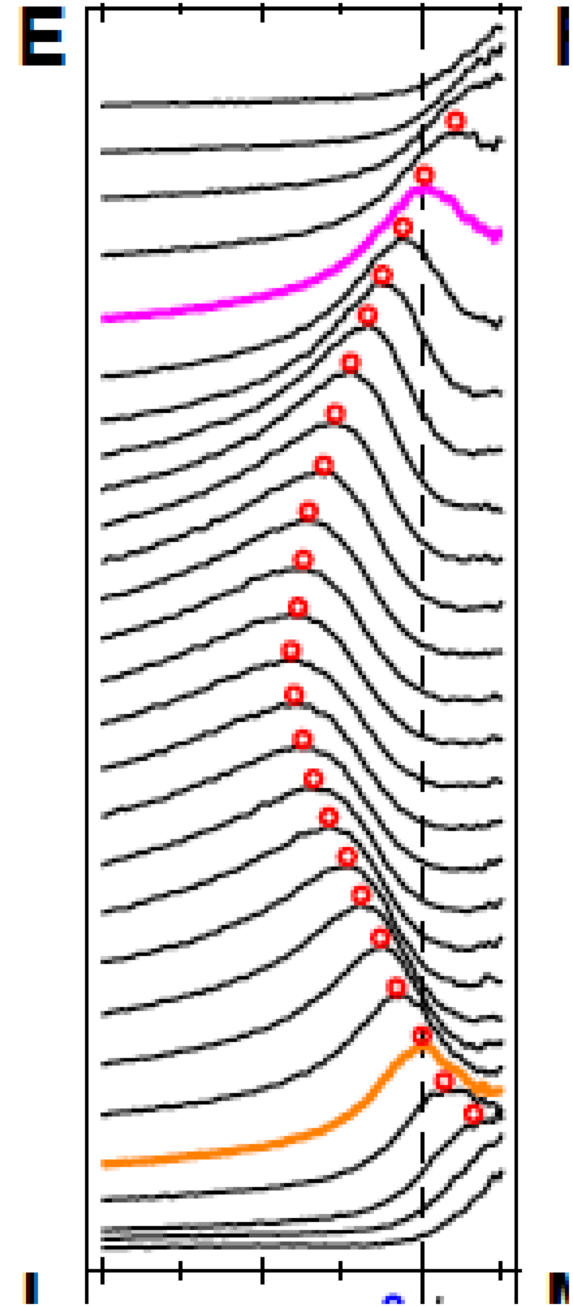


“Normal” state (T > T*) ARPES from
$Pb_{0.55}Bi_{1.5}Sr_{1.6}La_{0.4}CuO_{6+\delta}$

R-H He, M. Hashimoto, H. Karapetyan, J.C.Koralek, J. P. Hinton, J. . Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D.H. Lu, S-K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S.A.Kivelson, J. Orenstein, A. Kapitulnik, Z-X. Shen, Science **331**, 1579 (2011).

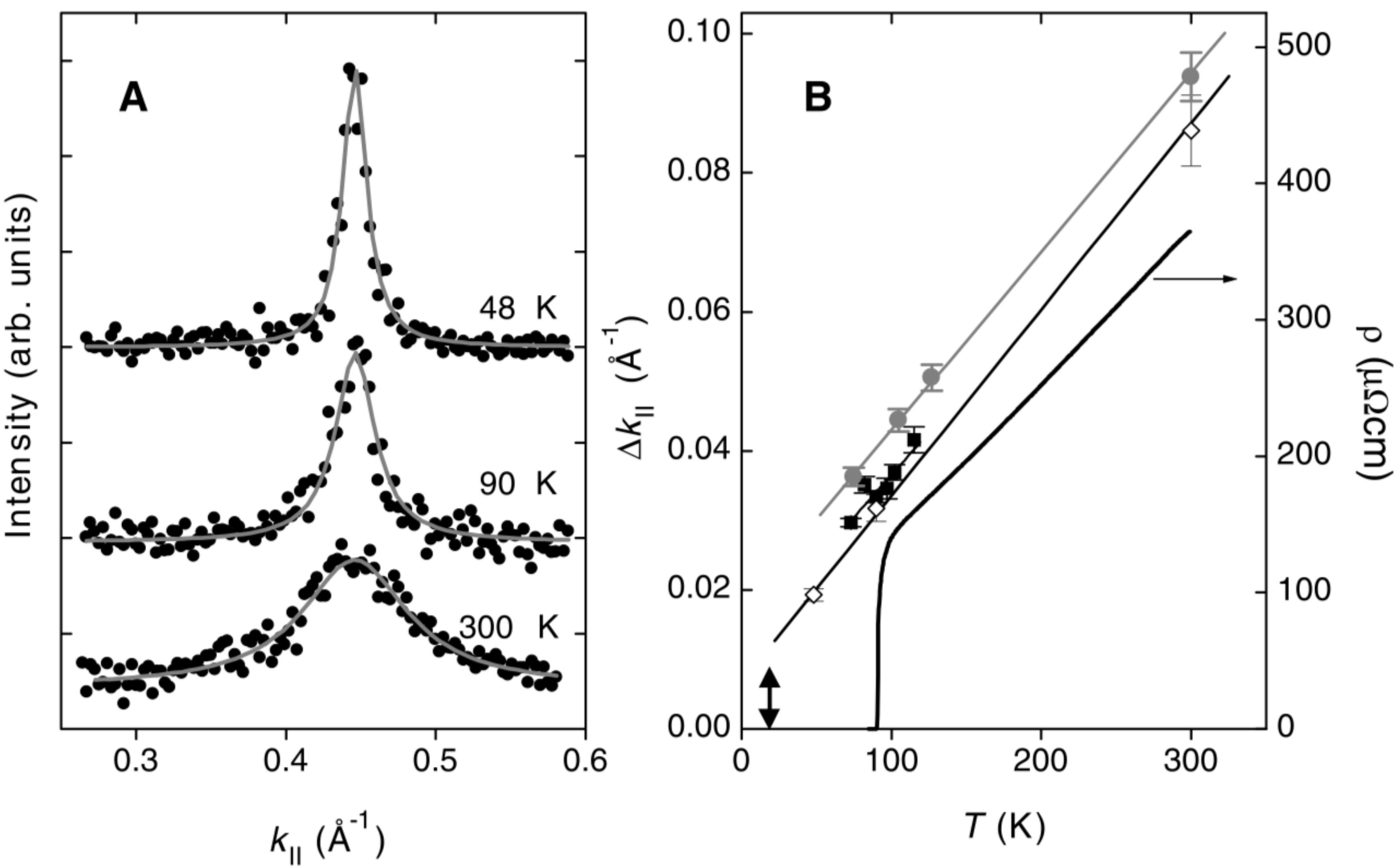
$$A(\vec{k},\omega) \propto I(\vec{k},\omega)/f(\hbar\omega)$$

$$\Delta E \sim 0.05eV \approx 500K$$



0.2eV 0eV

Evidence for quantum critical behavior in the optimally doped cuprate $Bi_2Sr_2CaCu_2O_{8+\delta}$,
T. Valla, A.V. Fedorov, P.D. Johnson et al, Science **285**, 2110 (1999).



Non-perturbative effects of electron-phonon coupling: Non-Boltzman Transport

Erez Berg and Yochai Weiman,

Y. Werman and E. Berg, “Mott-Ioffe-Regel limit and resistivity crossover in
A tractable electron-phonon model,” Phys. Rev. B 93, 075109 (2016).

Y. Werman, SAK and E. Berg, “Non-quasiparticle transport and resistivity
Saturation: a view from the large-N limit,” NPJ Quantum Materials 2, 7 (2017)

Summary

1) There exists a solvable model of a metal with strong electron-phonon coupling for $T >> T_c$

2) It exhibits a crossover from Boltzmann transport to semi-quantum transport when $1/\tau \sim E_F$

3) Depending on character of the electron-phonon coupling it either exhibits resistivity saturation or “bad metal” behavior.

In neither case is the notion of a maximum scattering rate applicable $\Gamma \sim [\lambda E_F T]^{1/2}$ for $E_F/\lambda << T << E_F$

4) It remains to see whether this has anything to do with the properties of ***real*** materials.