Characterization and measurement of qubit-environment entanglement generation during pure dephasing

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Abstract

The problem of detecting entanglement between a qubit and its environment is known to be complicated [1]. To simplify the issue, we study the class of Hamiltonians that describe a qubit interacting with its environment in such a way that the resulting evolution of the qubit alone is of pure dephasing type. Although this leads to some loss of generality, the pure dephasing Hamiltonian describes the dominant decohering mechanism for many types of qubits and is of fairly wide applicability. We define this situation by the requirement that the Hamiltonian of the qubit commutes with the qubit-environment interaction term. This relation means that the eigenstates of the qubit Hamiltonian form a preferred basis - they are pointer states [2,3] - selected by the form of the qubit-environment coupling. When both the qubit and the environment can initially be described by a (separable) wavefuntion (their state is pure throughout the evolution), an interaction between them that leads to a pure dephasing of the qubit always leads to the creation of entanglement between the two [4]. It is often assumed that a dephasing mechanism of this type must induce entanglement between the qubit and environment also when the environment is initially in a mixed state. We show that while the creation of qubit-environment entanglement in the pure dephasing case is possible when the environment is initially in a mixed state, the occurrence of this entanglement is by no means guaranteed.

We find that there are three types of situations (specified by the initial state of the environment and a relevant evolution operator which is derived from the Hamiltonian) when qubit-environment entanglement will not be generated. These are, the case when the initial density matrix of the environment is proportional to unity, the case when the relevant evolution operator cannot change the occupation of any of the eigenstates of the density matrix, and a non-trivial mixture of the two cases which allows dynamical evolution within closed subspaces of equal occupation.

Furthermore, we have shown that restricting the class of studied initial environmental states to a certain class of states (which is very common in any realistic qubit-environment setup) enables the use of a very powerful tool to measure the entanglement, since the state of the environment will remain static throughout the evolution (the state of the environment is found by tracing out the qubit degrees of freedom). Hence, the detection of any change of the state of the environment is then equivalent to the detection of entanglement.

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