

High-dimensional combinatorics – A progress report

Graph theory has been keeping Uzy busy for many years, but given his endless energy we should come up with new challenges for him so he cannot rest for another half century of research. I hope to infect him also with love of high-dimensional simplicial complexes. Our starting point is that a graph is a one-dimensional simplicial complex, so that many of the things that we know about graphs make sense for general simplicial complexes.

In this talk I will report on some of our discoveries in this domain. My main emphasis will be on a theory of random simplicial complexes that we have been investigating for several years now. Our methodology is to work vis-à-vis the theory of $G(n,p)$ graphs. In particular we report on discoveries that parallel the well known facts about the threshold for graph connectivity and about the phase transition at $p=1/n$.

My lecture is based on joint papers with Roy Meshulam, Lior Aronshtam, Tomasz Łuczak and Yuval Peled.