How many stable equilibria will a large complex system have?

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We aim to provide the quantitative answer to the classical question posed by Robert May (1972) "Will a Large Complex System be Stable?". To this end we analyse a generic autonomous nonlinear system of N>>1 randomly coupled ODE' describing degrees of freedom relaxing with the common rate \mu>0. We show that with decreasing rate \mu such systems experience an abrupt transition at some critical value \mu=\mu_C from a trivial phase portrait with a single stable equilibrium into a topologically non-trivial 'absolute instability' regime for \mu_B<\mu<\mu_C where equilibria are exponentially abundant, but typically all of them are unstable. Finally, at even smaller relaxation rate \mu<\mu_B stable equilibria become exponentially abundant, but their fraction to totality of all equilibria remains exponentially small. The revealed picture goes much beyond the May's linear analysis and is expected to be of relevance in the applications of complex systems to ecology, population biology, neural network theory and other areas. The presentation will be based on joint works with Gerard Ben Arous and Boris Khoruzhenko.