

# Classical and quantum aspects of higher-dimensional systems

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As the simplest example of higher-dimensional systems with a mixed phase space we consider 4D maps. The global organization of regular tori is visualized using 3D phase-space slices [1, 6]. By this we explain the more general hierarchy in 4D maps [2]. Bifurcations of families of 1D tori are present without parameter variation and play an important role in shaping the geometry of regular regions [3].

Quantum mechanically, the Husimi representation restricted to the 3D phase-space slice allows for comparing regular and chaotic eigenstates with classical structures to investigate the semiclassical eigenfunction hypothesis.

Such 4D maps can also be interpreted as two coupled 2D systems, i.e. an example of a bipartite system. If these two subsystems are strongly chaotic, we demonstrate that spectral statistics show a universal transition towards random matrix fluctuations for increasing interaction strength [4]. Moreover, entanglement in eigenstates, as measured by the von-Neumann entropy, shows a universal transition to nearly maximal entanglement [5].

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<http://www.comp-phys.tu-dresden.de/supp/>