GALOIS MEETS NEWTON: SYSTEMS OF EQUATIONS SOLVABLE BY RADICALS, AND NEWTON POLYTOPES OF VOLUME 4

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The classical Abel–Ruffini theorem claims that the general polynomial equation of degree d is solvable in radicals if and only if $d \leq 4$. I will present an analogue of this theorem for systems of equations: the square system of general polynomial equations with the Newton polytope N is solvable in radicals iff it has at most 4 solutions, i.e. the integer volume of N is at most 4. This in particular allows to classify all solvable systems. The only known proof involves recent strong results on Newton polytopes and on finite groups. The same question for a system of equations with non-equal Newton polytopes of equations is an open problem.