On polynomially integrable planar billiards

Alexey Glutsyuk

CNRS, ENS de Lyon (France) and Higher School of Economics (Russia)

The algebraic version of the famous Birkhoff Conjecture (partly investigated by S.V.Bolotin, M.Bialy and A.E.Mironov) deals with a billiard in a convex planar domain that admits a first integral polynomial in the speed vector. It states that *if the polynomial integral exists and is non-constant on the unit level hypersurface of the modulus of the speed, then the billiard table is an ellipse.* We present its solution that is a result of two works: 1) the work of M.Bialy and A.E.Mironov on angular billiards; 2) the recent work of the speaker, with a significant contribution of E.I.Shustin. We also present a solution of an analogous conjecture on the *outer billiards* stated and partially investigated by Sergei Tabachnikov: we show that *a polynomially integrable convex outer billiard is an ellipse.* This is a joint work with E.I.Shustin.

The proofs of both results use similar ideas and methods of complex singularity theory and algebraic geometry.

We give a survey of results on the general, non-algebraic Birkhoff conjecture on integrable planar billiards.

Email: aglutsyu@ens-lyon.fr