MILNOR NUMBER, INTERSECTION MULTIPLICITY AND NUMBER OF ZEROES OF SYSTEMS OF POLYNOMIALS

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Abstract

We talk about two of the original problems from the theory of Newton polyhedra: the problem of computing the Milnor number of the singularity at the origin of a generic polynomial, and computing the number of zeroes of generic polynomials. The former was addressed by Kushnirenko, who gave a beautiful formula in terms of Newton diagrams in a special case. Bernstein, following work of Kushnirenko, gave a complete solution of the latter problem for the case of $(\mathbb{C}^*)^n$, and the case of \mathbb{C}^n was addressed in works of Khovanskii, Huber-Sturmfels, and many others. We supply the ingredient missing from existing solutions of both these problems (of computing the generic Milnor number, and computing the generic number of solutions on \mathbb{C}^n), namely an explicit characterization of the systems for which the generic bound is achieved. The common thread to our approaches to these problems is the study of intersection multiplicity at the origin of the hypersurfaces determined by n generic polynomials.