LOGARITHMIC RESOLUTION OF SINGULARITIES

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The famous Hironaka's theorem asserts that any integral algebraic variety X of characteristic zero can be modified to a smooth variety X_{res} by a sequence of blowings up. The proof was simplified in subsequent works by Bierstone and Milman, Villamayor, Włodarczyk and others, and the obtained desingularization is canonical and compatible with smooth morphisms $X' \to X$ in the sense that $X'_{\mathrm{res}} = X_{\mathrm{res}} \times_X X'$.

The aim of my joint project with D. Abramovich and J. Włodarczyk is to achieve canonical desingularization of morphisms $Y \to X$ by so-called log smooth ones. Naturally, this algorithm should be compatible with all log-smooth morphisms, hence it must differ from the classical algorithm even in the case when Y is a variety and X is a point. At this stage we have already constructed the latter algorithm, that we call logarithmic desingularization of varieties, and we expect the same algorithm to apply to morphisms too.

In my talk I will describe the main ideas of the classical algorithm and show some examples. After that I will outline the main changes one should make in order to obtain the new algorithm. In brief, one should replace derivations, orders of ideals, the notion of smoothness, etc., by their logarithmic analogues. For example, desingularization is achieved by working with ideals on toroidal (or log smooth) varieties rather than on smooth ones. As a result, our algorithm is more technically demanding than the classical one, but it is faster and possesses better functoriality properties.