FLEXIBLE POLYHEDRA IN SPACES OF CONSTANT CURVATURE AND THEIR VOLUMESY

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Consider an oriented closed polyhedral surface in 3-dimensional Euclidean space whose faces are rigid plates and adjacent faces are connected by hinges at edges. If this polyhedral surface admits a non-trivial (i. e., not induced by a rotation of the ambient space) deformation, then it is called a *flexible polyhedron*.

One of the most interesting facts concerning flexible polyhedra is the *bellows conjecture* asserting that the volume of any polyhedron is constant during the flexion. This conjecture was proved by Sabitov in 1996, and the proof is based on the following result that is important in itself: The volume of any simplicial polyhedron in 3-dimensional Euclidean space satisfies a monic polynomial relation with coefficients being polynomials in edge lengths of the polyhedron. Similarly, we can consider flexible polyhedra in Euclidean spaces of dimensions greater than 3, and also in spaces of non-zero constant curvature, that is, in spherical spaces and in Lobachevsky spaces. The latter result remains true in Euclidean spaces of higher dimensions (the author, 2012). However, it is by no means true in non-Euclidean spaces of constant curvature. In fact, even in the simplest case of a tetrahedron in 3-dimensional Lobachevsky space the function expressing the volume of a tetrahedron from its edge lengths is a very complicated transcendental function.

In the present talk, we shall discuss recent results by the author on the bellows conjecture in non-Euclidean spaces. These results are based on the study of the analytic continuation of the function that expresses the volume from the cosines (or hyperbolic cosines) of the edge lengths of the polyhedron.