

REAL CURVES AND NON-COMMUTATIVE HOMOTOPY GROUPS IN TORIC SURFACES

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To real curves of type I intersecting toric divisors only at real points one can associate a half-integer number called the quantum index. This number comes as the signed area enclosed in \mathbb{R}^2 by the logarithmic image of the real curve divided by π^2 (surprisingly, it turns out that the result is always half-integer). The quantum index can be used to refine enumeration of real curves in toric surfaces from numbers to q -numbers.

In the talk we'll explore a different, homotopy, viewpoint on the quantum index of rational real curves. Namely, a certain second relative homotopy group associated to a toric surface is a non-commutative group of Heisenberg type. Rational real curves realize elements of this non-commutative group while the quantum index is given by the Heisenberg homomorphism.

Joint work with Sergey Galkin.