

DIFFERENTIAL EQUATIONS AND ALGEBRAIC POINTS ON TRANSCENDENTAL VARIETIES

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The problem of bounding the number of rational or algebraic points of a given height in a transcendental set has a long history. In 2006 Pila and Wilkie made fundamental progress in this area by establishing a sub-polynomial asymptotic estimate for a very wide class of transcendental sets. This result plays a key role in Pila-Zannier's proof of the Manin-Mumford conjecture, Pila's proof of the Andre-Oort conjecture for modular curves, Masser-Zannier's work on torsion anomalous points in elliptic families, and many more recent developments.

I will briefly sketch the Pila-Wilkie theorem and the way it enters into the arithmetic applications. I will then discuss recent work on an effective form of the Pila-Wilkie theorem (for certain sets) which leads to effective versions of many of the applications. I will also discuss a joint work with Dmitry Novikov on sharpening the asymptotics from sub-polynomial to poly-logarithmic for certain structures, leading to a proof of the restricted Wilkie conjecture. The structure of the systems of differential equations satisfied by various transcendental functions plays a key role for both of these directions.