

## Wayne StatE UNIVERSITY

The Proton Radius Puzzle

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# Introduction: The proton radius puzzle 

- Matrix element of EM current between nucleon states give rise to two form factors $\left(q=p_{f}-p_{i}\right)$

$$
\left\langle N\left(p_{f}\right)\right| \sum_{q} e_{q} \bar{q} \gamma^{\mu} q\left|N\left(p_{i}\right)\right\rangle=\bar{u}\left(p_{f}\right)\left[\gamma^{\mu} F_{1}\left(q^{2}\right)+\frac{i \sigma_{\mu \nu}}{2 m} F_{2}\left(q^{2}\right) q^{\nu}\right] u\left(p_{i}\right)
$$

- Sachs electric and magnetic form factors

$$
\begin{array}{ll}
G_{E}\left(q^{2}\right)=F_{1}\left(q^{2}\right)+\frac{q^{2}}{4 m_{p}^{2}} F_{2}\left(q^{2}\right) & G_{M}\left(q^{2}\right)=F_{1}\left(q^{2}\right)+F_{2}\left(q^{2}\right) \\
G_{E}^{p}(0)=1 & G_{M}^{p}(0)=\mu_{p} \approx 2.793
\end{array}
$$

- The slope of $G_{E}^{P}$

$$
\left\langle r^{2}\right\rangle_{E}^{p}=\left.6 \frac{d G_{E}^{p}}{d q^{2}}\right|_{q^{2}=0}
$$

determines the charge radius $r_{E}^{p} \equiv \sqrt{\left\langle r^{2}\right\rangle_{E}^{p}}$

- The proton magnetic radius

$$
\left\langle r^{2}\right\rangle_{M}^{p}=\left.\frac{6}{G_{M}^{p}(0)} \frac{d G_{M}^{p}\left(q^{2}\right)}{d q^{2}}\right|_{q^{2}=0}
$$

## Charge radius from atomic physics

$$
\left\langle p\left(p_{f}\right)\right| \sum_{q} e_{q} \bar{q} \gamma^{\mu} q\left|p\left(p_{i}\right)\right\rangle=\bar{u}\left(p_{f}\right)\left[\gamma^{\mu} F_{1}^{p}\left(q^{2}\right)+\frac{i \sigma_{\mu \nu}}{2 m} F_{2}^{p}\left(q^{2}\right) q^{\nu}\right] u\left(p_{i}\right)
$$

- For a point particle amplitude for $p+\ell \rightarrow p+\ell$

$$
\mathcal{M} \propto \frac{1}{q^{2}} \Rightarrow U(r)=-\frac{Z \alpha}{r}
$$

- Including $q^{2}$ corrections from proton structure

$$
\mathcal{M} \propto \frac{1}{q^{2}} q^{2}=1 \quad \Rightarrow \quad U(r)=\frac{4 \pi Z \alpha}{6} \delta^{3}(r)\left(r_{E}^{p}\right)^{2}
$$

- Proton structure corrections $\left(m_{r}=m_{\ell} m_{p} /\left(m_{\ell}+m_{p}\right) \approx m_{\ell}\right)$

$$
\Delta E_{r_{E}^{p}}=\frac{2(Z \alpha)^{4}}{3 n^{3}} m_{r}^{3}\left(r_{E}^{p}\right)^{2} \delta_{\ell 0}
$$

- Muonic hydrogen can give the best measurement of $r_{E}^{p}$ !


## Charge radius from atomic physics nature <br> 

- Lamb shift in muonic hydrogen [Pohl et al. Nature 466, 213 (2010)] $r_{E}^{p}=0.84184(67) \mathrm{fm}$
more recently $\mathrm{r}_{E}^{p}=0.84087(39) \mathrm{fm}$ [Antognini et al. Science 339, 417 (2013)]
- CODATA value [Mohr et al. RMP 80, 633 (2008)] $r_{E}^{p}=0.87680(690) \mathrm{fm}$ more recently $r_{E}^{p}=0.87510(610) \mathrm{fm}$ [Mohr et al. RMP 88, 035009 (2016)] extracted mainly from (electronic) hydrogen
- $5 \sigma$ discrepancy!
- This is the proton radius puzzle


## Great outreach opportunity!

- Problem easily communicated to general audience
- Example: Detroit high school students using data

[R. Pohl et al., "The size of the proton," Nature 466, 213 (2010)]
and the approximate formula, $f=50.59 \mathrm{THz}-r^{2} \frac{\mathrm{THz}}{\mathrm{fm}^{2}}$
to determine $r=0.84 \mathrm{fm}$


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1) Problem with the electronic extraction? (Part 1 of this talk)
2) Misunderstood proton structure effects? (Part 2 of this talk)
3) New Physics?

## Outline

- Introduction: The proton radius puzzle
- Part 1: Proton radii from scattering
- Part 2: Hadronic Uncertainty?
- Conclusions and outlook


## Part 1: Proton radii from scattering

## Problem with the electronic extraction?

- You can get the proton radius from electron-proton scattering
- Recent development: use of the $z$ expansion based on known analytic properties of form factors [Hill, GP PRD 82113005 (2010)]
- The method for meson form factors [Flavor Lattice Averaging Group, EPJ C 74, 2890 (2014)]
- Now applied successfully to baryon form factors to extract $r_{E}^{p}, r_{M}^{p}, r_{M}^{n}, m_{A} \ldots$


## Form Factors: What we do know

- Analytic properties of $G_{E}^{p}(t)$ and $G_{M}^{p}(t)$ are known
- They are analytic outside a cut $t \in\left[4 m_{\pi}^{2}, \infty\right]$
[Federbush, Goldberger, Treiman, Phys. Rev. 112, 642 (1958)]
- $e-p$ scattering data is in $t<0$ region

- If your form factor doesn't have this analytic structure it's wrong! (e.g. singularity at $4 m_{\pi}^{2}$ : why should the Taylor series converge?)


## z expansion

- z expansion:

We can map the domain of analyticity onto the unit circle

$$
z\left(t, t_{\mathrm{cut}}, t_{0}\right)=\frac{\sqrt{t_{\mathrm{cut}}-t}-\sqrt{t_{\mathrm{cut}}-t_{0}}}{\sqrt{t_{\mathrm{cut}}-t}+\sqrt{t_{\mathrm{cut}}-t_{0}}}
$$

where $t_{\text {cut }}=4 m_{\pi}^{2}, z\left(t_{0}, t_{\text {cut }}, t_{0}\right)=0$


- Expand $G_{E, M}^{p}$ in a Taylor series in z: $G_{E, M}^{p}\left(q^{2}\right)=\sum_{k=0}^{\infty} a_{k} z\left(q^{2}\right)^{k}$


## $z$ expansion

- [Zachary Epstein, GP, Joydeep Roy PRD 90, 074027 (2014)]


## z expansion

- [Zachary Epstein, GP, Joydeep Roy PRD 90, 074027 (2014)] $G_{M}\left(Q^{2}\right)$ for proton (blue, above axis) and neutron (red, below axis)



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$G_{M}(z)$ for proton (blue, above axis) and neutron (red, below axis)

- See also R.J. Hill talk at FPCP 2006 [hep-ph/0606023]


## PDG 2016: $r_{E}^{p}$

Citation: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016)

## p CHARGE RADIUS

This is the rms electric charge radius, $\sqrt{\left\langle r_{E}^{2}\right\rangle}$.

| VALUE (fm) | DOCUMENT ID |  | TECN | COMMENT |
| :---: | :---: | :---: | :---: | :---: |
| $0.8751 \pm 0.0061$ | MOHR | 16 | RVUE | 2014 CODATA value |
| $0.84087 \pm 0.00026 \pm 0.00029$ | ANTOGNINI | 13 | LASR | $\mu p$-atom Lamb shift |
| - We do not use the following data for averages, fits, limits, etc. - |  |  |  |  |
| $0.895 \pm 0.014 \pm 0.014$ | 1 LEE | 15 | SPEC | Just 2010 Mainz data |
| $0.916 \pm 0.024$ | LEE | 15 | SPEC | World data, no Mainz |
| $0.8775 \pm 0.0051$ | MOHR | 12 | RVUE | 2010 CODATA, ep data |
| $0.875 \pm 0.008 \pm 0.006$ | ZHAN | 11 | SPEC | Recoil polarimetry |
| $0.879 \pm 0.005 \pm 0.006$ | BERNAUER | 10 | SPEC | $e p \rightarrow$ ep form factor |
| $0.912 \pm 0.009 \pm 0.007$ | BORISYUK | 10 |  | reanalvzes old ep data |
| $0.871 \pm 0.009 \pm 0.003$ | HILL | 10 |  | z-expansion reanalvsis |
| $0.84184 \pm 0.00036 \pm 0.00056$ | POHL | 10 | LASR | See ANTOGNINI 13 |
| $0.8768 \pm 0.0069$ | MOHR | 08 | RVUE | 2006 CODATA value |
| $\begin{array}{ll}0.844 & +0.008 \\ & -0.004\end{array}$ | BELUSHKIN | 07 |  | Dispersion analysis |
| $0.897 \pm 0.018$ | BLUNDEN | 05 |  | SICK $03+2 \gamma$ correction |
| $0.8750 \pm 0.0068$ | MOHR | 05 | RVUE | 2002 CODATA value |
| $0.895 \pm 0.010 \pm 0.013$ | SICK | 03 |  | $e p \rightarrow e p$ reanalysis |
| [Hill, GP PRD 82113005 (2010)] |  |  |  |  |
| [Lee, Arrington, Hill, PRD 92, 013013 (2015)] |  |  |  |  |

## PDG 2016: $r_{M}^{p}$

Citation: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016)

## p MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\left\langle r_{M}^{2}\right\rangle}$.


-     - We do not use the following data for averages, fits, limits, etc.

| $0.914 \pm 0.035$ | LEE | 15 | SPEC | World data, no Mainz |
| :--- | :--- | :--- | :--- | :--- |
| $0.87 \pm 0.02$ | EPSTEIN | 14 |  | Using ep, en, $\pi \pi$ data |
| $0.867 \pm 0.009 \pm 0.018$ | ZHAN | 11 | SPEC | Recoil polarimetry |
| $0.777 \pm 0.013 \pm 0.010$ | BERNAUER | 10 | SPEC | $e p \rightarrow$ ep form factor |
| $0.876 \pm 0.010 \pm 0.016$ | BORISYUK | 10 |  | Reanalyzes old ep $\rightarrow$ ep data |
| $0.854 \pm 0.005$ | BELUSHKIN | 07 |  | Dispersion analysis |

${ }^{1}$ Authors also provide values for a combination of all available data.

> [Epstein, GP, Roy PRD 90, 074027 (2014)]
> [Lee, Arrington, Hill, PRD 92, 013013 (2015)]

Citation: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016)

## n MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\left\langle r_{M}^{2}\right\rangle}$.

| $\underline{\text { VALUE }(\mathrm{fm})}$ |  | DOCUMENT ID |  | COMMENT |
| :--- | :--- | :--- | :--- | :--- |
| $\mathbf{0 . 8 6 4} \mathbf{+ 0 . 0 0 9}$ | OUR AVERAGE |  |  |  |
| $0.89 \pm 0.03$ |  |  |  |  |
| $0.862_{-0.008}^{+0.009}$ | EPSTEIN | 14 | Using e $p$, e $n, \pi \pi$ data |  |

$$
\text { [Epstein, GP, Roy PRD 90, } 074027 \text { (2014)] }
$$

# Part 2: Hadronic Uncertainty? 

[Hill, GP PRD 95, 094017 (2017), arXiv:1611.09917]

## The bottom line

- Scattering:
- World $e-p$ data [Lee, Arrington, Hill '15] $r_{E}^{p}=0.918 \pm 0.024 \mathrm{fm}$
- Mainz e-p data [Lee, Arrington, Hill '15]
$r_{E}^{p}=0.895 \pm 0.020 \mathrm{fm}$
- Proton, neutron and $\pi$ data [Hill, GP '10] $r_{E}^{p}=0.871 \pm 0.009 \pm 0.002 \pm 0.002 \mathrm{fm}$
- Muonic hydrogen
- [Pohl et al. Nature 466, 213 (2010)] $r_{E}^{p}=0.84184(67) \mathrm{fm}$
- [Antognini et al. Science 339, 417 (2013)] $r_{E}^{p}=0.84087(39) f m$
- The bottom line: using $z$ expansion scattering disfavors muonic hydrogen
- Is there a problem with muonic hydrogen theory?
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- Potentially yes! [Hill, GP PRL 107160402 (2011)]
- The proton radius arises from one photon probe
- Increasing precision requires also a two photon probe
a much more complicated object


## Muonic hydrogen theory

- Is there a problem with muonic hydrogen theory?
- Potentially yes!
[Hill, GP PRL 107160402 (2011)]
- Muonic hydrogen measures $\Delta E$ and translates it to $r_{E}^{p}$
- [Pohl et al. Nature 466, 213 (2010) Supplementary information] $\Delta E=206.0573(45)-5.2262\left(r_{E}^{p}\right)^{2}+0.0347\left(r_{E}^{p}\right)^{3} \mathrm{meV}$
- [Antognini et al. Science 339, 417 (2013), Ann. of Phy. 331, 127] $\Delta E=206.0336(15)-5.2275(10)\left(r_{E}^{p}\right)^{2}+0.0332(20) \mathrm{meV}$
- In both cases apart from $r_{E}^{p}$ need two-photon exchange

- Apart from $r_{E}^{p}$ we have two-photon exchange (TPE)

- Imaginary part of TPE related to data: form factors, structure functions


## Two photon exchange

- Apart from $r_{E}^{p}$ we have two-photon exchange (TPE)

- Imaginary part of TPE related to data: form factors, structure functions
- Cannot reproduce it from its imaginary part:

Dispersion relation requires subtraction

- Need poorly constrained non-perturbative function $W_{1}\left(0, Q^{2}\right)$

Two photon exchange

- Apart from $r_{E}^{p}$ we have two-photon exchange (TPE)


$$
\begin{aligned}
W^{\mu \nu}= & \frac{1}{2} \sum_{s} i \int d^{4} x e^{i q \cdot x}\langle\boldsymbol{k}, s| T\left\{J_{\text {e.m. }}^{\mu}(x) J_{\text {e.m. }}^{\nu}(0)\right\}|\boldsymbol{k}, s\rangle \\
& =\left(-g^{\mu \nu}+\frac{q^{\mu} q^{\nu}}{q^{2}}\right) W_{1}+\left(k^{\mu}-\frac{k \cdot q q^{\mu}}{q^{2}}\right)\left(k^{\nu}-\frac{k \cdot q q^{\nu}}{q^{2}}\right) W_{2}
\end{aligned}
$$

- Dispersion relations $\left(\nu=2 k \cdot q, Q^{2}=-q^{2}\right)$

$$
\begin{aligned}
& W_{1}\left(\nu, Q^{2}\right)=W_{1}\left(0, Q^{2}\right)+\frac{\nu^{2}}{\pi} \int_{\nu_{\mathrm{cut}}\left(Q^{2}\right)^{2}}^{\infty} d \nu^{\prime 2} \frac{\operatorname{Im} W_{1}\left(\nu^{\prime}, Q^{2}\right)}{\nu^{\prime 2}\left(\nu^{\prime 2}-\nu^{2}\right)} \\
& W_{2}\left(\nu, Q^{2}\right)=\frac{1}{\pi} \int_{\nu_{\mathrm{cut}}\left(Q^{2}\right)^{2}}^{\infty} d \nu^{\prime 2} \frac{\operatorname{Im} W_{2}\left(\nu^{\prime}, Q^{2}\right)}{\nu^{\prime 2}-\nu^{2}}
\end{aligned}
$$

- $W_{1}$ requires subtraction...
- Small $Q^{2}$ limit using NRQED [Hill, GP, PRL 107160402 (2011)] The photon sees the proton "almost" like an elementary particle $W_{1}\left(0, Q^{2}\right)=2 a_{p}\left(2+a_{p}\right)+\frac{Q^{2}}{m_{p}^{2}}\left\{\frac{2 m_{p}^{3} \bar{\beta}}{\alpha}-a_{p}-\frac{2}{3}\left[\left(1+a_{p}\right)^{2} m_{p}^{2}\left(r_{M}^{p}\right)^{2}-m_{p}^{2}\left(r_{E}^{p}\right)^{2}\right]\right\}+\mathcal{O}\left(Q^{4}\right)$

$$
W_{1}\left(0, Q^{2}\right)=13.6+\frac{Q^{2}}{m_{p}^{2}}(-54 \pm 7)+\mathcal{O}\left(Q^{4}\right)
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- $\mathcal{O}\left(Q^{4}\right)$ depend on unmeasured higher dim. NRQED matrix elements [Gunawardna, GP JHEP 1707137 (2017), Kobach, Pal PLB 772225 (2017)]
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Two Photon Exchange: large $Q^{2}$ limit

- Large $Q^{2}$ limit using Operator Product Expansion (OPE) The photon "sees" the quarks and gluons inside the proton

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W_{1}\left(0, Q^{2}\right)=c / Q^{2}+\mathcal{O}\left(1 / Q^{4}\right)
$$

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RENORMALIZATION OF THE COTTINGHAM FORMULA
John C. COLLINS *
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA

Received 23 October 1978

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- Collins confirmed the mistake in [J. C. Collins, NPB 915, 392 (2017)]


## Large $Q^{2}$ behavior



- Performing the complete calculation, we found a mistake in Collins spin-0 calculation from 1978...
- Collins didn't calculate the spin-0 gluon contribution directly He extracted it from another calculation
- For three light quark $u, d, s$

Correct result: $\sum_{q} e_{q}^{2}=\left(\frac{2}{3}\right)^{2}+\left(\frac{1}{3}\right)^{2}+\left(\frac{1}{3}\right)^{2}=\frac{2}{3}$
Collins: $\sum_{q}=3$
Too large by a factor of 4.5...

## Two Photon Exchange: Modeling

- Simple modeling: use OPE for $Q^{2} \geq 1 \mathrm{GeV}^{2}$
- Model unknown $Q^{4}$ : add $\Delta_{L}\left(Q^{2}\right)= \pm Q^{2} / \Lambda_{L}^{2}$ with $\Lambda_{L} \approx 500 \mathrm{MeV}$
- Model unknown $1 / Q^{4}$ : add $\Delta_{H}\left(Q^{2}\right)= \pm \Lambda_{H}^{2} / Q^{2}$ with $\Lambda_{H} \approx 500 \mathrm{MeV}$


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- How to connect the curves?



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- Interpolating:

- Energy contribution: $\delta E(2 S)^{W_{1}\left(0, Q^{2}\right)} \in[-0.046 \mathrm{meV},-0.021 \mathrm{meV}]$ To explain the puzzle need this to be $\sim-0.3 \mathrm{meV}$
- Caveats: OPE might be only valid for larger $Q^{2}$ $W_{1}\left(0, Q^{2}\right)$ might be different than the interpolated lines


## Two Photon Exchange: Other approaches

- Similar results found by other groups

[34] K. Pachucki, PRA 60, 3593 (1999).
[35] A. P. Martynenko, Phys. At. Nucl. 69, 1309 (2006).
[36] D. Nevado and A. Pineda, PRC 77, 035202 (2008).
[33] C. E. Carlson and M. Vanderhaeghen, PRA 84, 020102 (2011).
[3] M. C. Birse and J. A. McGovern, EPJA 48, 120 (2012).
[37] Gorchtein, Llanes-Estrada, Szczepaniak, PRA 87, 052501 (2013).
[38] J. M. Alarcon, V. Lensky, and V. Pascalutsa, EPJC 74, 2852 (2014).
[5] C. Peset and A. Pineda, Nucl. Phys. B887, 69 (2014).
[4] Antognini, Kottmann, Biraben, Indelicato, Nez, Pohl, Ann. Phys. 331, 127 (2013).
[Fig. 8] Hill, GP PRD 95, 094017 (2017).


## Experimental test

- How to test? New experiment: $\mu-p$ scattering MUSE (MUon proton Scattering Experiment) at PSI [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]

- Need to connect muon-proton scattering and muonic hydrogen can use a new effective field theory: QED-NRQED
[Hill, Lee, GP, Mikhail P. Solon, PRD 87053017 (2013)]
[Steven P. Dye, Matthew Gonderinger, GP, PRD 94013006 (2016)]
[Steven P. Dye, Matthew Gonderinger, GP, in progress]


## Conclusions

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- Proton radius puzzle: $>5 \sigma$ discrepancy between
- $r_{E}^{p}$ from muonic hydrogen
- $r_{E}^{p}$ from hydrogen and $e-p$ scattering


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- Much more work to do!
- Thank you

